A General Exact Formulation for the Outage Probability in Interference-Limited Systems

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***Abstract*—This paper presents a useful, novel formulation for the outage probability in interference-limited communication systems, here named *Joint Outage Probability* (JOP). Given a set of SIR restrictions for mutually interfering signals, the JOP corresponds to the probability that at least one of the restrictions is not satisfied. A general exact solution for the joint outage probability is derived, along with a necessary and sufficient condition for a non-null JOP. Furthermore, a closed-form expression for the joint outage probability in a non-identically distributed Rayleigh scenario with independent signals is obtained. The results presented here can be directly applied in a wide range of practical scenarios.**

***Index Terms*—Outage Probability, Signal-to-Interference Ratio, Interference-Limited Systems, Rayleigh.**

I. Introduction

Outage probability is a key figure of merit in wireless communications. In interference-limited systems, outage probability is commonly defined as the probability that the signal-to-interference ratio (SIR) of a received signal is below a given threshold [1], [2]. Such a metric is directly related to system capacity in spread spectrum schemes, such as CDMA systems [3]. Furthermore, dense wireless networks, such as sensor and ad hoc networks, are intrinsically interference-limited, having the outage probability as a fundamental parameter for system analysis, design, and implementation [4]–[6].

The difficulty of the analysis of an outage condition may vary drastically. In some situations, it may be as simple as obtaining the probability of the occurrence of a single, straightforward event. In some others, it may involve the calculation of the probability of joint, intricate events. For instance, in call admission problems it may be desirable to have a call admitted if, with such an admission, the interference experienced by all of the calls (i.e. those in progress as well as the one entering the system) remains under a certain tolerable threshold. In case the channels are independent and identically distributed (i.i.d.) and the threshold conditions are the same for all of the conversations (communications), one may lay hold of a symmetry property to simplify this problem and find an approximate solution as follows. Considering each channel to be identically affected by all of the others, then it approximately suffices to test the outage condition of any arbitrary individual channel with the admission of the call. But as said, this is an approximation even to the i.i.d. case. In addition, in practical situations and in multi-rate, multi-service networks, the channels are affected differently and the interference threshold may vary for each communication. Therefore, an exact solution for this general case is of paramount interest.

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In light of the previous discussion, we present a useful, novel formulation for interference-limited communication systems, here named *Joint Outage Probability* (JOP). Given a set of SIR restrictions for N mutually interfering signals with a certain joint distribution, the JOP corresponds to the probability that at least one of the restrictions is not satisfied or, dually, that all N restrictions are attended. Hence, the JOP gives a direct measure of the outage probability for a group of signals detected by a receiver, not being limited to the outage of an individual signal. Although the results presented here have a theoretical nature, they can be directly applied in a wide range of practical scenarios, such as multiuser detection [7], power control [8], sensor positioning, wireless (multihop or not) network dimensioning [9] and, as hinted earlier, admission control problems [10]. To the best of the authors' knowledge, such a result is unprecedented in the literature.

The remainder of this paper is organized as follows. Section II presents a formal definition of the JOP problem. Section III derives the general, exact formulation to solve the JOP. In addition, a necessary and sufficient condition for a non-trivial JOP, dependent only on the SIR restrictions, and not on the signals' distribution, is found. Section IV presents a closed-form expression for the JOP in an independent non-identically Rayleigh distributed scenario with arbitrary interference thresholds. Finally, Section V presents some concluding remarks and a summary of the results.

II. Outline of the Problem

Let W_i, $i = 1, \ldots, N$ be the instantaneous powers of interfering fading signals. In an interference-limited system, in order for the system to operate adequately, it is required that the signal-to-interference ratio at the receiver for any given signal W_i be greater than a tolerable threshold β_i, $i = 1, \ldots, N$, as specified for the particular communication.

Therefore, the N received signals must satisfy the set of inequalities (1), which describes a hypervolume in the N-dimensional space $\mathcal{S}^N$.

$$\mathcal{S}^N = \begin{cases} \dfrac{W_1}{\sum_{i=2}^{N} W_i} \geq \beta_1 \\ \dfrac{W_2}{\sum_{\substack{i=1\\i\neq 2}}^{N} W_i} \geq \beta_2 \\ \quad\vdots \\ \dfrac{W_N}{\sum_{i=1}^{N-1} W_i} \geq \beta_N \end{cases} \tag{1}$$

We define as Joint Outage Probability (P_I) the probability that at least one of the inequalities in (1) is not satisfied. The value of P_I can be obtained as

$$P_{\bar{I}} = \int_{\mathcal{S}^N} f_{\mathbf{W}}(w_1, \ldots, w_N)\; dw_1 \ldots dw_N \ , \tag{2}$$

where $f_{\mathbf{W}}(w_1, \ldots, w_N)$ is the joint probability density function of $W_1, \ldots, W_N$ and $P_{\bar{I}} = 1 - P_I$ is the probability that all the inequalities in (1) are satisfied.

III. The Exact Solution for the JOP

In this section, the region $\mathcal{S}^N$ will be reformulated so as to make the integration in (2) tractable. This is done through four steps: (i) $\mathcal{S}^N$ will be divided into $N-1$ regions, denoted by $\mathcal{S}_j^N$, $j = 1, \ldots, N-1$, each of which with well-defined integration limits; (ii) the union $\bigcup_{j=1}^{N-1} \mathcal{S}_j^N$ is shown to be equivalent to $\mathcal{S}^N$; (iii) $\mathcal{S}_i^N$ and $\mathcal{S}_j^N$ ($j \neq i$) are proved to form distinct integration regions; (iv) as a consequence of the previous steps, the right side of (2) will then be rewritten as the sum of $N-1$ integrals. The section is then finalized by determining the condition for a non-trivial $\mathcal{S}^N$ hypervolume.

A. Dividing $\mathcal{S}^N$ into $N-1$ Regions

The restrictions in (1) can be rearranged in terms of the W_N as:

$$\begin{cases} W_N \leq \dfrac{W_1}{\beta_1} - \sum_{i=2}^{N-1} W_i \\ W_N \leq \dfrac{W_2}{\beta_2} - \sum_{\substack{i=1\\i\neq 2}}^{N-1} W_i \\ \quad\vdots \\ W_N \leq \dfrac{W_{N-1}}{\beta_{N-1}} - \sum_{i=1}^{N-2} W_i \\ W_N \geq \beta_N \sum_{i=1}^{N-1} W_i \end{cases} \tag{3}$$

Noting that the first $N-1$ inequalities and the last inequality in (3) determine, respectively, upper and lower bounds for the value of W_N, (3) can be rewritten as

$$\beta_N \sum_{i=1}^{N-1} W_i \leq W_N \leq \min_{1\leq k\leq N-1} \left(\frac{W_k}{\beta_k} - \sum_{\substack{i=1\\i\neq k}}^{N-1} W_i \right) \ . \tag{4}$$

(Of course, from (3) and (4), null values of β_k will not impose an upper limit to W_N. With this observation in mind, and without loss of generality, we shall consider $\beta_k \neq 0$ for all k.)

Analyzing the upper limit for W_N in (4), we find that $k = j$, $1 \leq j \leq N-1$, when

$$W_j \leq \frac{\beta_j(1+\beta_i)}{\beta_i(1+\beta_j)} W_i \quad \forall i < N \ . \tag{5}$$

Consider a region such that (5) holds. Then (4) is rewritten as

$$\beta_N \sum_{i=1}^{N-1} W_i \leq W_N \leq \frac{W_j}{\beta_j} - \sum_{\substack{i=1\\i\neq j}}^{N-1} W_i \ . \tag{6}$$

A necessary condition for (6) to be non-trivial is

$$\frac{(1-\beta_j\beta_N)}{\beta_j(1+\beta_N)} W_j \geq \sum_{\substack{i=1\\i\neq j}}^{N-1} W_i \ . \tag{7}$$

For convenience, we define the constants

$$\mathbb{K}_{j,k} = \frac{\beta_j(1+\beta_k)}{\beta_k(1+\beta_j)} \tag{8}$$

$$\mathbb{C}_{j,N} = \frac{(1-\beta_j\beta_N)}{\beta_j(1+\beta_N)} \ . \tag{9}$$

Combining (5) to (7), the following restriction is found for W_{N-1}:

$$\mathbb{K}_{N-1,j} W_j \leq W_{N-1} \leq \mathbb{C}_{j,N} W_j - \sum_{\substack{i=1\\i\neq j}}^{N-2} W_i \ . \tag{10}$$

In the same way, for W_{N-2},

$$\mathbb{K}_{N-2,j} W_j \leq W_{N-2} \leq \mathbb{C}_{j,N} W_j - \sum_{\substack{i=1\\i\neq j}}^{N-3} W_i - W_{N-1} \ . \tag{11}$$

Using the lower bound for W_{N-1} in (10), (11) can be rewritten as

$$\mathbb{K}_{N-2,j} W_j \leq W_{N-2} \leq (\mathbb{C}_{j,N} - \mathbb{K}_{N-1,j}) W_j - \sum_{\substack{i=1\\i\neq j}}^{N-3} W_i \ . \tag{12}$$

Proceeding in a similar manner for $W_{N-3}, W_{N-4}, \ldots, W_1$, we find the following set of inequalities, which describe a region defined as $\mathcal{S}_j^N$:

$$\begin{cases} \beta_N \sum_{i=1}^{N-1} W_i \le W_N \le \dfrac{W_j}{\beta_j} - \sum_{\substack{i=1 \\ i\neq j}}^{N-1} W_i \\ \mathbb{K}_{N-1,j} W_j \le W_{N-1} \le \mathbb{C}_{j,N} W_j - \sum_{\substack{i=1 \\ i\neq j}}^{N-2} W_i \\ \quad\vdots \qquad\qquad \vdots \\ \mathbb{K}_{k,j} W_j \le W_k \le (\mathbb{C}_{j,N} - \sum_{\substack{i=k+1 \\ i\neq j}}^{N-1} \mathbb{K}_{i,j}) W_j - \sum_{\substack{i=1 \\ i\neq j}}^{k-1} W_i \\ \quad\vdots \qquad\qquad \vdots \\ \mathbb{K}_{1,j} W_j \le W_1 \le (\mathbb{C}_{j,N} - \sum_{\substack{i=2 \\ i\neq j}}^{N-1} \mathbb{K}_{i,j}) W_j \\ 0 \le W_j < \infty \end{cases} \tag{13}$$

B. Proof that $\bigcup_{j=1}^{N-1} \mathcal{S}_j^N = \mathcal{S}^N$

Next, we prove that the region formed by the union $\bigcup_{j=1}^{N-1} \mathcal{S}_j^N$ is equal to that of $\mathcal{S}^N$. From the derivation of $\mathcal{S}_j^N$, it is clear that an N-tuple $(W_1, W_2, \ldots, W_N)$ belonging to $\mathcal{S}^N$, with the jth element, $1 \le j \le N-1$, satisfying (5), will also be contained in $\mathcal{S}_j^N$. We note that, in case $(W_1, W_2, \ldots, W_N) \in \mathcal{S}^N$, there will always be at least one element that satisfies (5), since the upper limit of (4) necessarily has a minimum. If m elements satisfy (5), it is straightforward to show that the N-tuple will belong to m regions of the form $\mathcal{S}_j^N$. It follows that any given N-tuple belonging to $\mathcal{S}^N$ will also be contained in $\bigcup_{j=1}^{N-1} \mathcal{S}_j^N$.

The converse can be proved by contradiction. Assume that an N-tuple $(W_1, W_2, \ldots, W_N)$ belonging to $\bigcup_{j=1}^{N-1} \mathcal{S}_j^N$ exists that does not satisfy at least one of the inequalities in (3). Then, we may consider that, for some $k \neq N$,

$$W_N > \frac{W_k}{\beta_k} - \sum_{\substack{i=1 \\ i\neq k}}^{N-1} W_i \ . \tag{14}$$

In case W_k satisfies (5), (14) confutes the first inequality in (13), contradicting the initial assumption. Otherwise, suppose, without loss of generality, that condition (5) holds for $j \neq k$. Thus,

$$\begin{aligned} \frac{W_k}{\beta_k} &< W_N + \sum_{\substack{i=1 \\ i\neq k}}^{N-1} W_i \\ &\le \frac{W_j}{\beta_j} - \sum_{\substack{i=1 \\ i\neq j}}^{N-1} W_i + \sum_{\substack{i=1 \\ i\neq k}}^{N-1} W_i \\ \Rightarrow W_k &< \mathbb{K}_{k,j} W_j \ , \end{aligned}$$

contradicting the fact that W_j satisfies (5).

When $k = N$, we have

$$W_N < \beta_N \sum_{i=1}^{N-1} W_i \ ,$$

contradicting the first inequality in (13). Therefore, an element belonging to the region $\bigcup_{j=1}^{N-1} \mathcal{S}_j^N$ will also belong to $\mathcal{S}^N$.

C. Proof that $\mathcal{S}_j^N$ and $\mathcal{S}_k^N$ Form Distinct Integration Regions

We will prove that the N-dimensional hypervolume of $\mathcal{S}_j^N \cap \mathcal{S}_k^N$ ($j \neq k$) is zero, implying that $\mathcal{S}_j^N$ and $\mathcal{S}_k^N$ are distinct integration regions. In case an element $(W_1, W_2, \ldots, W_N)$ belongs to both $\mathcal{S}_j^N$ and $\mathcal{S}_k^N$, we find from (5) that

$$W_j = \mathbb{K}_{j,k} W_k \ . \tag{15}$$

Since two elements are dependent, the fact that the N-dimensional hypervolume is null follows directly. Intuitively, this means that the regions $\mathcal{S}_j^N$ and $\mathcal{S}_k^N$ have only "border" points in common, with disjoint interiors. Therefore, the integration region defined by $\mathcal{S}_j^N$ does not overlap with the one defined by $\mathcal{S}_k^N$.

D. Main Result

Based on the previous developments, (2) can be rewritten as

$$P_{\bar{I}} = \sum_{j=1}^{N-1} \int_{\mathcal{S}_j^N} f_{\mathbf{W}}(w_1, \ldots, w_N) \; dw_1 \ldots dw_N \ , \tag{16}$$

or, equivalently,

$$\begin{aligned} P_{\bar{I}} = \sum_{j=1}^{N-1} \int_0^\infty \int_{\mathbb{K}_{1,j} w_j}^{(\mathbb{C}_{j,N} - \sum_{\substack{i=2 \\ i\neq j}}^{N-1} \mathbb{K}_{i,j}) w_j} \cdots \\ \times \int_{\mathbb{K}_{j-1,j} w_j}^{(\mathbb{C}_{j,N} - \sum_{i=j+1}^{N-1} \mathbb{K}_{i,j}) w_j - \sum_{i=1}^{j-2} w_i} \int_{\mathbb{K}_{j+1,j} w_j}^{(\mathbb{C}_{j,N} - \sum_{i=j+2}^{N-1} \mathbb{K}_{i,j}) w_j - \sum_{i=1}^{j-1} w_i} \cdots \\ \times \int_{\mathbb{K}_{N-1,j} w_j}^{\mathbb{C}_{j,N} w_j - \sum_{\substack{i=1 \\ i\neq j}}^{N-2} w_i} \int_{\beta_N \sum_{i=1}^{N-1} w_i}^{\frac{w_j}{\beta_j} - \sum_{\substack{i=1 \\ i\neq j}}^{N-1} w_i} f_{\mathbf{W}}(w_1, w_2, \ldots, w_N) \\ \times \; dw_N dw_{N-1} \ldots dw_{j+1} dw_{j-1} \ldots dw_1 dw_j \ . \end{aligned} \tag{17}$$

We note that the index j of the summation terms in (16) and (17) will only span the values for which $\beta_j \neq 0$.

E. The Condition for a Non-Trivial $\mathcal{S}^N$

From (17), a necessary and sufficient condition for the integration region defined in (1) to be a non-trivial one is

$$\sum_{i=1}^{N} \frac{\beta_i}{1+\beta_i} < 1 \ . \tag{18}$$

This is proved as follows. In order for (13) to define a non-trivial integration region, the upper limit must be necessarily larger than the lower limit for every inequality. Therefore, from the restriction for W_1 in (13) (or equivalently for W_2 in case $\mathcal{S}_1^N$ is considered), we have that

$$\mathbb{C}_{j,N} > \sum_{\substack{i=1\\i\neq j}}^{N-1} \mathbb{K}_{i,j} \ . \tag{19}$$

We will proceed by induction to show that (19) is also a sufficient condition for the integration region $\mathcal{S}_j^N$ to be non-trivial. Assume that (19) is satisfied, which is the base case of the inductive argument. Suppose, also, that the upper limit of the inequality corresponding to W_k in (13) is necessarily larger than the lower one. Since the integration region for W_k is non-trivial and considering, without loss of generality, $k \neq j-1$ and W_k strictly smaller than its upper bound in (13), it follows that

$$W_k < (\mathbb{C}_{j,N} - \sum_{\substack{i=k+1\\i\neq j}}^{N-1} \mathbb{K}_{i,j})W_j - \sum_{\substack{i=1\\i\neq j}}^{k-1} W_i$$
$$\Rightarrow \sum_{\substack{i=1\\i\neq j}}^{k} W_i < (\mathbb{C}_{j,N} - \sum_{\substack{i=k+1\\i\neq j}}^{N-1} \mathbb{K}_{i,j})W_j \ . \tag{20}$$

Analyzing the upper limit for $k+1$:

$$(\mathbb{C}_{j,N} - \sum_{\substack{i=k+2\\i\neq j}}^{N-1} \mathbb{K}_{i,j})W_j - \sum_{\substack{i=1\\i\neq j}}^{k} W_i > \mathbb{K}_{k+1,j} W_j \ . \tag{21}$$

The inequality follows directly from (20), proving the induction step. Thus, if the condition for W_1 is satisfied in (13), the restriction for $W_2,\ W_3, ...,\ W_{N-1}$ will be non-trivial.

Finally, to prove that W_N will also have a non-trivial restriction when (19) is satisfied, we note that its limits in (13) depend on all the other $N-1$ variables. Consequently, it must be shown that $W_1,\ W_2, \ldots,\ W_{N-1}$ can assume values such that

$$\mathbb{C}_{j,N} W_j > \sum_{\substack{i=1\\i\neq j}}^{N-1} W_i \ , \tag{22}$$

where the inequality follows from (7). However, given (19), (22) will be satisfied when $W_1,\ W_2, \ldots,\ W_{N-1}$ assume their minimum values in (13), which concludes the proof. The restriction (18) follows directly by summing $1 + \mathbb{K}_{N,j}$ to both sides of (19).

It is interesting to note that (18) is independent of $\mathcal{S}_j^N$, even though a specific region was considered in the proof. Furthermore, (18) depends solely on the SIR thresholds, and not on the signals' distribution. This fact explicits that, even though the inequalities in (1) may have a non-null probability of being satisfied individually, there will always be at least one signal outage unless (18) is attended.

IV. General, Closed-Form Expression for a Rayleigh Environment

In this section, a closed-form expression for the joint outage probability in a Rayleigh environment is derived, assuming independent interfering signals. The probability density function of W_i is given by

$$f_{W_i}(w_i) = \frac{1}{\Omega_i} \exp\left(-\frac{w_i}{\Omega_i}\right) \ , \tag{23}$$

where Ω_i is the mean power. The joint density distribution of $W_1, W_2, \ldots, W_N$ is

$$f_{\mathbf{W}}(w_1, w_2, , \ldots, w_N) = \prod_{i=1}^{N} f_{W_i}(w_i) \ . \tag{24}$$

The integration (17) of the joint distribution (24) presents a regularity, as a consequence of being performed over the product of exponential functions. Hence, for a fixed j in the summation term, the innermost integrals of (17) have a pattern, denoted by $\varphi_j(\cdot)$. Thus, $\varphi_j(\cdot)$ satisfies the following two conditions, expressed in terms of $j=1$ in (17):

$$\varphi_1(r, s, \mathbf{\Omega}, \boldsymbol{\beta}, 1) = \int_{\mathbb{K}_{N-1,1} r}^{\mathbb{C}_{1,N} r - s} \frac{e^{-\frac{t}{\Omega_{N-1}}}}{\Omega_{N-1}} \int_{\beta_N (r+s+t)}^{\frac{r}{\beta_1} - s - t} \frac{e^{-\frac{u}{\Omega_N}}}{\Omega_N} du dt \tag{25}$$

and, for $1 < k < N-1$:

$$\varphi_1(r, s, \mathbf{\Omega}, \boldsymbol{\beta}, k) = \int_{\mathbb{K}_{N-k,1} r}^{(\mathbb{C}_{1,N} - \sum_{i=N-k+1}^{N-1} \mathbb{K}_{i,1}) r - s} \frac{e^{-\frac{t}{\Omega_{N-k}}}}{\Omega_{N-k}} \times \varphi(r, s+t, \mathbf{\Omega}, \boldsymbol{\beta}, k-1) dt \ , \tag{26}$$

where r and s are auxiliary variables, respectively, denoting w_j and the sum of the remaining w_i which will be posteriorly integrated, $\mathbf{\Omega} = (\Omega_1, \Omega_2, \ldots, \Omega_N)$ and $\boldsymbol{\beta} = (\beta_1, \beta_2, \ldots, \beta_N)$.

The function $\varphi_1(\cdot)$ is given by

$$\varphi_1(r, s, \mathbf{\Omega}, \boldsymbol{\beta}, c) = \sum_{i=N-c}^{N} \frac{(1+\beta_N)\Omega_N \Omega_i^c}{(\beta_N \Omega_i + \Omega_N) \prod_{\substack{j=N-c\\j\neq i}}^{N} (\Omega_i - \Omega_j)}$$
$$\times \left(\exp\left(-\frac{(r+s)\beta_N}{\Omega_N} - r \sum_{j=N-c}^{N-1} \frac{(\Omega_N + \beta_N \Omega_j)}{\Omega_j \Omega_N} \mathbb{K}_{j,1} \right) - \exp\left(\frac{\beta_i s - r}{\beta_i \Omega_i} + r \sum_{\substack{j=N-c\\j\neq i}}^{N} \frac{(\Omega_j - \Omega_i)}{\Omega_j \Omega_i} \mathbb{K}_{j,1} \right) \right) \ , \tag{27}$$

where $1 \leq c < N-1$. It is straightforward to show that (27) satisfies (25) and (26).

The value of $\varphi_i(\cdot)$ for $i \neq 1$, as well as the corresponding properties (25) and (26), can be obtained directly from (27) with the appropriate change of indexes. When two signals W_m and W_n have the same mean power, the corresponding expression for $\varphi_1(\cdot)$ can be found by taking the limit $\Omega_m \to \Omega_n$ in (27).

Using (17) and the property (26) of $\varphi_i(\cdot)$, the value of $P_{\bar{I}}$ for the Rayleigh scenario under investigation is

$$P_{\bar{I}} = \sum_{m=1}^{N-1} P_{\bar{I},m} \ , \tag{28}$$

where

$$P_{\bar{I},m} = \int_0^\infty \frac{e^{-\frac{r}{\Omega_m}}}{\Omega_m} \varphi_m(r, 0, \mathbf{\Omega}, \boldsymbol{\beta}, N-2)\; dr\ . \qquad (29)$$

Evaluating the integral in (29), and after tedious and lengthy simplifications, $P_{\bar{I},m}$ is found to be

$$P_{\bar{I},m} = \frac{\left(\frac{(1+\beta_N)\beta_m}{1+\beta_m}\right)\left(1-\sum_{j=1}^{N}\frac{\beta_j}{1+\beta_j}\right)^{N-1}}{\left(\prod_{l=1}^{N}\Omega_l\right)\left(\sum_{j=1}^{N-1}\frac{\beta_j}{1+\beta_j}\left(\frac{\beta_N\Omega_j+\Omega_N}{\Omega_j\Omega_N}\right)\right)} \times \prod_{\substack{i=1\\ i\neq m}}^{N}\left(\frac{1}{\Omega_i}+\sum_{j=1}^{N}\frac{\beta_j\left(\Omega_i-\Omega_j\right)}{\left(1+\beta_j\right)\Omega_i\Omega_j}\right)^{-1}\ . \qquad (30)$$

Substituting (30) in (28) and performing further simplifications, we arrive at the final exact, closed-form expression for $P_{\bar{I}}$:

$$P_{\bar{I}} = \frac{\left(1-\sum_{j=1}^{N}\frac{\beta_j}{1+\beta_j}\right)^{N-1}}{\prod_{i=1}^{N}\left(1-\sum_{j=1}^{N}\frac{\beta_j}{1+\beta_j}\left(1-\frac{\Omega_i}{\Omega_j}\right)\right)}\ . \qquad (31)$$

This expression is general, and may be applied even when one or more signals have the same average power or a null restriction value, i.e., $\beta_j = 0$. When only one signal has a non-null SIR threshold value, corresponding to the case when the outage of a single user is considered, (31) reduces to the expressions found elsewhere in the literature (e.g. [11], [12]). It is noteworthy that condition (18) appears explicitly in the numerator of (31).

When $\beta_j = \beta$ and $\Omega_j = \Omega$ for all j, (31) simplifies to

$$P_{\bar{I}} = \left(\frac{1-(N-1)\beta}{1+\beta}\right)^{N-1}\ . \qquad (32)$$

Equation (32) is illustrated in Fig. 1.

V. Conclusions

In this paper, we presented a new formulation for interference-limited wireless systems, namely joint outage probability. An integral form for the JOP was derived in terms of the individual SIR restrictions and the signals' joint probability distribution. Through further analysis, a necessary and sufficient condition for a non-null JOP was found, which depends solely on the SIR restrictions. An application of these results led to an *exact*, simple, closed-form expression of the joint outage probability in a Rayleigh environment, considering independent signals. The formulations derived here are general, and may be applied in various practical scenarios.

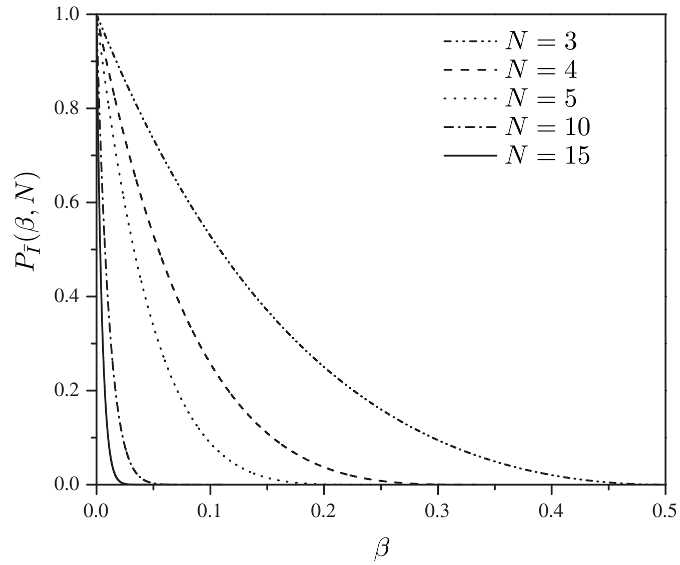

Fig. 1. $P_{\bar{I}}$ for a Rayleigh i.i.d. environment with equal restrictions β.

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