

Correspondence

General Exact Level Crossing Rate and Average Fade Duration for Dual-Diversity Combining of Nonidentical Correlated Weibull Signals

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Abstract—This paper derives general exact expressions for the level crossing rate (LCR) and the average fade duration (AFD) of dual-branch selection, equal-gain, and maximal-ratio combiners operating over nonidentical correlated Weibull fading channels. Sample numerical results are discussed by specializing the general expressions to a space-diversity system using horizontally spaced antennas at a mobile station. It is verified that as the antenna spacing becomes larger, the LCR decreases, becoming oscillatory and convergent. In addition, when the direction of the mobile is perpendicular to the axis of the antenna, the AFD is loosely dependent on the antenna spacing. Some simulation results are presented to verify the correctness of the analytical formulation.

Index Terms—Average fade duration (AFD), diversity-combining techniques, level crossing rate (LCR), Weibull fading channels.

I. INTRODUCTION

Diversity-combining techniques constitute an effective means to combat the deleterious effects of multipath fading on the performance of wireless communication systems. This performance can be evaluated by several measures, including the level crossing rate (LCR) and the average fade duration (AFD). Although the branch signals may be correlated and nonidentically distributed in practical systems [1]–[6], the literature on the LCR and AFD of diversity techniques over nonidentical correlated fading is not as rich as for the independent scenario. Pioneering work on this issue was carried out by Adachi *et al.* [1] for dual-branch selection combining (SC), equal-gain combining (EGC), and maximal-ratio combining (MRC) over balanced correlated Rayleigh channels. The unbalanced correlated Rayleigh and Ricean cases were addressed in [2] and [3] for MRC. In [4], Yang *et al.* presented a unified treatment for LCR and AFD of M -branch SC over unbalanced correlated Rayleigh, Ricean, and Nakagami- m channels. In [5], the LCR and AFD for the MRC were derived for a correlated unbalanced Nakagami- m environment. More recently, an extension of [1] for unbalanced channels was

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investigated in [6]. To the best of the authors' knowledge, these second-order statistics for correlated nonidentical Weibull fading channels have not been investigated in the literature yet. This paper derives general exact expressions for the LCR and AFD for dual-branch SC, EGC, and MRC combining systems in a Weibull fading environment. The expressions apply to nonidentical correlated diversity channels. Some numerical results are presented for a space-diversity system using horizontally spaced antennas at a mobile station. To verify the correctness of the analytical formulation, simulation data are also provided.

This paper is organized as follows: Section II establishes the model for the Weibull fading channels and derives the Weibull joint bidimensional envelope-phase density (JBEPD). Some key statistics involving the branch envelopes and their time derivatives are derived in Section III. Relying upon these statistics, general exact LCR and AFD expressions are also presented. Section IV computes the conditional means and variances for each diversity system. Section V shows some numerical and simulation plots, and Section VI draws some conclusions. Appendix A details the formulation of the complex covariance matrix. Appendix B demonstrates the relation between the conditional statistics (means, variances, and covariance) of the real variates with those of the complex variates.

II. PRELIMINARIES

The Weibull distribution is an empirical distribution, which was first proposed aiming at applications in reliability engineering. It has also found use in wireless communications to model the fading envelope [7]–[9]. In [10] and [11], a very simple physical model for the Weibull distribution was proposed. In essence, in the proposed model, the received signal Z_i at branch i ($i = 1, 2$) can be represented in a complex form as

$$Z_i = R_i^{\alpha_i/2} \exp(j\Theta_i) = X_i + jY_i \quad (1)$$

where $\sqrt{j} = -1$; R_i is the Weibull envelope; Θ_i is the Weibull phase, which is uniformly distributed in $[0, 2\pi)$; X_i and Y_i are independent zero-mean Gaussian variates with identical variances σ_i^2 ; and $\alpha_i > 0$ stands for the Weibull fading parameter. The probability density function (pdf) $f_{R_i}(\cdot)$ of the envelope R_i is given by

$$f_{R_i}(r_i) = \frac{\alpha_i r_i^{\alpha_i-1}}{\Omega_i} \exp\left(-\frac{r_i^{\alpha_i}}{\Omega_i}\right) \quad (2)$$

where $\Omega_i = E(R_i^{\alpha_i}) = 2\sigma_i^2$, and $E(\cdot)$ stands for the statistical average. For the special cases $\alpha_i = 1$ and $\alpha_i = 2$, (2) reduces to the negative exponential and Rayleigh pdfs, respectively. The k th moment of R_i is expressed as

$$E(R_i^k) = \Omega_i^{k/\alpha_i} \Gamma\left(1 + \frac{k}{\alpha_i}\right). \quad (3)$$

The mean power P_i is obtained directly from (3) by setting $k = 2$. From (1), it can be seen that the Weibull envelope R_i is obtained as the modulus of the multipath Rayleigh envelope R_{R_i} to the power $2/\alpha_i$. Hence, the relation between R_i and R_{R_i} can be expressed as $R_i = R_{R_i}^{2/\alpha_i}$.

The Weibull JBEPD $f_{R_1, R_2, \Theta_1, \Theta_2}(r_1, r_2, \theta_1, \theta_2)$ is attained by capitalizing on the results available in the literature for Rayleigh distribution. The Rayleigh JBEPD $f_{R_{R_1}, R_{R_2}, \Theta_1, \Theta_2}(\cdot, \cdot, \cdot, \cdot)$ is given in [12, eq. (7.51)]. Based on the relation between R_i and R_{R_i} , as described above, it follows that $R_1^{\alpha_1} = R_{R_1}^2$, and $R_2^{\alpha_2} = R_{R_2}^2$. Of course, $f_{R_1, R_2, \Theta_1, \Theta_2}(r_1, r_2, \theta_1, \theta_2) = |J| f_{R_{R_1}, R_{R_2}, \Theta_1, \Theta_2}(\cdot, \cdot, \cdot, \cdot)$, where $|\cdot|$ denotes the determinant operator, and J is the Jacobian of the transformation. From the above, $|J| = (\alpha_1 \alpha_2 / 4) r_1^{\alpha_1/2-1} r_2^{\alpha_2/2-1}$. Using this and after algebraic manipulations, we have

$$\begin{aligned} & f_{R_1, R_2, \Theta_1, \Theta_2}(r_1, r_2, \theta_1, \theta_2) \\ &= \frac{\alpha_1 \alpha_2 r_1^{\alpha_1-1} r_2^{\alpha_2-1}}{4\pi^2 \Omega_1 \Omega_2 (1-\rho^2)} \exp\left(-\frac{r_1^{\alpha_1} \Omega_2 + r_2^{\alpha_2} \Omega_1}{(1-\rho^2) \Omega_1 \Omega_2}\right) \\ & \times \exp\left(\frac{2r_1^{\frac{\alpha_1}{2}} r_2^{\frac{\alpha_2}{2}} \sqrt{\Omega_1 \Omega_2} \rho \cos(\theta_2 - \theta_1 - \phi)}{(1-\rho^2) \Omega_1 \Omega_2}\right) \end{aligned} \quad (4)$$

where $\rho^2 = [E^2(X_1 X_2) + E^2(X_1 Y_2)] / \sigma_1^2 \sigma_2^2$, and $\phi = \text{Arg}[E(X_1 X_2) + jE(X_2 Y_1)]$.

III. LCR AND AFD

The LCR $N_R(r)$ and the AFD $T_R(r)$ of a random signal are defined, respectively, as the average number of upward (or downward) crossings per second at a given level and as the mean time the signal remains below this level after crossing it in the downward direction. The LCR and AFD of the output combiner $R = R(t)$ at level r are, respectively, given by

$$N_R(r) = \int_0^\infty \dot{r} f_{R, \dot{R}}(r, \dot{r}) d\dot{r} \quad (5)$$

$$T_R(r) = \frac{F_R(r)}{N_R(r)} \quad (6)$$

where $f_{R, \dot{R}}(\cdot, \cdot)$ is the joint pdf of R and its time derivative \dot{R} , and $F_R(\cdot)$ is the cumulative distribution function (CDF) of the envelope R . In the following, (5) and (6) shall be calculated for a dual-branch nonidentical correlated Weibull fading environment using the SC, EGC, and MRC techniques.

A. Diversity Systems

The output envelope time derivative for the SC, EGC, and MRC combining systems is given by

$$\dot{R} = \begin{cases} \dot{R}_1, & R_1 \geq R_2 \\ \dot{R}_2, & R_1 < R_2 \end{cases}, \quad \text{SC} \\ \frac{\dot{R}_1 + \dot{R}_2}{\sqrt{2}}, \quad \text{EGC} \\ \frac{R_1 \dot{R}_1 + R_2 \dot{R}_2}{\sqrt{R_1^2 + R_2^2}}, \quad \text{MRC} \end{cases} \quad (7)$$

For all of the combining schemes, the density of \dot{R} given R_i 's and Θ_i 's, i.e., $f_{\dot{R}|R_1, R_2, \Theta_1, \Theta_2}(\dot{r}|r_1, r_2, \theta_1, \theta_2)$, is Gaussian with mean $m_{\dot{R}}(r_1, r_2, \theta_1, \theta_2) \triangleq m_{\dot{R}}$ and variance $\sigma_{\dot{R}}^2(r_1, r_2, \theta_1, \theta_2) \triangleq \sigma_{\dot{R}}^2$, as correctly and pioneeringly demonstrated in [1] and then in [6] for the Rayleigh case. These quantities depend on the combining scheme and shall be determined next. Now, using the properties of the conditional probability, the following can be written:

$$\begin{aligned} & f_{\dot{R}, R_1, R_2, \Theta_1, \Theta_2}(\dot{r}, r_1, r_2, \theta_1, \theta_2) \\ &= f_{\dot{R}|R_1, R_2, \Theta_1, \Theta_2}(\dot{r}|r_1, r_2, \theta_1, \theta_2) f_{R_1, R_2, \Theta_1, \Theta_2}(r_1, r_2, \theta_1, \theta_2) \end{aligned} \quad (8)$$

where $f_{R_1, R_2, \Theta_1, \Theta_2}(r_1, r_2, \theta_1, \theta_2)$ is given by (4). The tricky part of the problem is the determination of $m_{\dot{R}}$ and $\sigma_{\dot{R}}^2$ for each combining scheme. For the moment, assume that these quantities are known. Then, by means of [1, eq. (8)] for SC and of [13, eqs. (12) and (17)] for EGC and MRC, respectively, the LCR can be expressed as (9), shown at the bottom of the page, in which

$$\begin{aligned} \vartheta(r_1, r_2) &\triangleq \int_0^\infty \dot{r} f_{\dot{R}|R_1, R_2, \Theta_1, \Theta_2}(\dot{r}|r_1, r_2, \theta_1, \theta_2) d\dot{r} \\ &= \frac{\sigma_{\dot{R}}^2}{\sqrt{2\pi}} \exp\left(-\frac{m_{\dot{R}}}{2\sigma_{\dot{R}}^2}\right) + \frac{m_{\dot{R}}}{2} \left(1 + \text{erf}\left(\frac{m_{\dot{R}}}{\sqrt{2}\sigma_{\dot{R}}}\right)\right) \end{aligned} \quad (10)$$

where $\text{erf}(\cdot)$ is the error function. The CDF $F_R(\cdot)$ of R can be expressed as [14]

$$F_R(r) = \int_0^{\gamma_1} \int_0^{\gamma_2} \int_0^{2\pi} \int_0^{2\pi} f_{R_1, R_2, \Theta_1, \Theta_2}(r_1, r_2, \theta_1, \theta_2) d\theta_1 d\theta_2 dr_2 dr_1 \quad (11)$$

where

$$\begin{cases} \gamma_1 = \gamma_2 = r, & \text{for SC} \\ \gamma_1 = \sqrt{2}r, \quad \gamma_2 = \sqrt{2}r - r_1, & \text{for EGC} \\ \gamma_1 = r, \quad \gamma_2 = \sqrt{r^2 - r_1^2}, & \text{for MRC} \end{cases} \quad (12)$$

The AFD follows directly from (6), (9), and (11).

From (7), for each combining scenario, it is possible to express the mean $m_{\dot{R}}$ and variance $\sigma_{\dot{R}}^2$ of the conditional Gaussian density of \dot{R} in terms of the mean $m_{\dot{R}_i}$, variance $\sigma_{\dot{R}_i}^2$, and covariance $\sigma_{\dot{R}_i, \dot{R}_l}$ ($i, l = 1, 2$) of the conditional Gaussian density of \dot{R}_i given R_i 's and Θ_i 's, i.e., $f_{\dot{R}_i|R_1, R_2, \Theta_1, \Theta_2}(\dot{r}_i|r_1, r_2, \theta_1, \theta_2)$, based on the linearity of the mean operator [15, eq. (6-162)] and using the property given in [15, eq. (6-167)]. A similar procedure was also used in [1] but for correlated balanced Rayleigh fading channels.

$$N_R(r) = \begin{cases} \int_0^{2\pi} \int_0^{2\pi} \int_0^r \vartheta(r_1, r) f_{R_1, R_2, \Theta_1, \Theta_2}(r_1, r, \theta_1, \theta_2) dr_1 d\theta_1 d\theta_2 + \int_0^{2\pi} \int_0^{2\pi} \int_0^r \vartheta(r, r_2) f_{R_1, R_2, \Theta_1, \Theta_2}(r, r_2, \theta_1, \theta_2) dr_2 d\theta_1 d\theta_2 & \text{SC} \\ \int_0^{2\pi} \int_0^{2\pi} \int_0^{\sqrt{2}r} \sqrt{2} \vartheta(r_1, \sqrt{2}r - r_1) f_{R_1, R_2, \Theta_1, \Theta_2}(r_1, \sqrt{2}r - r_1, \theta_1, \theta_2) dr_1 d\theta_1 d\theta_2, & \text{EGC} \\ \int_0^{2\pi} \int_0^{2\pi} \int_0^r \frac{r}{\sqrt{r^2 - r_1^2}} \vartheta\left(r_1, \sqrt{r^2 - r_1^2}\right) f_{R_1, R_2, \Theta_1, \Theta_2}\left(r_1, \sqrt{r^2 - r_1^2}, \theta_1, \theta_2\right) dr_1 d\theta_1 d\theta_2, & \text{MRC} \end{cases} \quad (9)$$

IV. CONDITIONAL STATISTICS OF \dot{R}_i

This section constitutes a crucial step for solving the problem addressed here. The mean $m_{\dot{R}_i}$, variance $\sigma_{\dot{R}_i}^2$, and covariance $\sigma_{\dot{R}_i, \dot{R}_l}$, as required in the previous formulations, shall be obtained. Differentiating (1) with respect to time, it follows that $\dot{R}_i = (2/\alpha_i)R_i^{1-(\alpha_i/2)}\text{Re}[\dot{Z}_i \exp(-j\Theta_i)]$, where $\text{Re}[\cdot]$ denotes the real part of a complex number. The conditional statistics of \dot{R}_i 's can be written in terms of the conditional statistics of \dot{Z}_i 's as (see Appendix B)

$$\begin{aligned} m_{\dot{R}_i} &\triangleq \mathbb{E}(\dot{R}_i|\mathbf{Z}) \\ &= \frac{2}{\alpha_i} R_i^{1-\frac{\alpha_i}{2}} \text{Re} \left[\mathbb{E}(\dot{Z}_i|\mathbf{Z}) e^{-j\Theta_i} \right] \end{aligned} \quad (13)$$

$$\begin{aligned} \sigma_{\dot{R}_i}^2 &\triangleq \text{Var}(\dot{R}_i|\mathbf{Z}) \\ &= \frac{2R_i^{2-\alpha_i}}{\alpha_i^2} \text{Var}(\dot{Z}_i|\mathbf{Z}) \end{aligned} \quad (14)$$

$$\begin{aligned} \sigma_{\dot{R}_i, \dot{R}_l} &\triangleq \text{Cov}(\dot{R}_i, \dot{R}_l|\mathbf{Z}) \\ &= \frac{2}{\alpha_i \alpha_l} R_i^{1-\frac{\alpha_i}{2}} R_l^{1-\frac{\alpha_l}{2}} \text{Re} \left[e^{j\Theta_{il}} \text{Cov}(\dot{Z}_i, \dot{Z}_l|\mathbf{Z}) \right] \end{aligned} \quad (15)$$

where $\text{Var}(\cdot)$ and $\text{Cov}(\cdot)$ denote the variance and covariance, respectively. Define $\dot{\mathbf{Z}} = [\dot{Z}_1 \dot{Z}_2]$, and $\mathbf{Z} = [Z_1 Z_2]$, in which \dot{Z}_1 , \dot{Z}_2 , Z_1 , and Z_2 are mutually correlated zero-mean complex Gaussian variables. The complex covariance matrix Φ is defined as [1]

$$\Phi = \frac{1}{2} \mathbb{E} \left[\begin{pmatrix} \dot{\mathbf{Z}} \\ \mathbf{Z} \end{pmatrix}^* \begin{pmatrix} \dot{\mathbf{Z}} \\ \mathbf{Z} \end{pmatrix}^T \right] \triangleq \begin{bmatrix} \mathbf{a} & \mathbf{c} \\ \mathbf{c}^H & \mathbf{b} \end{bmatrix} \quad (16)$$

where $(\cdot)^*$ denotes the conjugate operator, $(\cdot)^T$ denotes the transpose matrix, and $(\cdot)^H$ denotes the Hermitian matrix. Defining $\rho_{il}(\tau)$ as the complex cross-correlation function between the i th and l th branches, then

$$\rho_{il}(\tau) = \frac{\mathbb{E}(Z_i^*(t)Z_l(t+\tau))}{\sqrt{\Omega_i \Omega_l}}. \quad (17)$$

The matrices \mathbf{a} , \mathbf{b} , and \mathbf{c} can be expressed as (see Appendix A)

$$\mathbf{a} = \frac{1}{2} \begin{bmatrix} -\dot{\rho}_{11}\Omega_1 & -\dot{\rho}_{12}\sqrt{\Omega_1\Omega_2} \\ -\dot{\rho}_{12}\sqrt{\Omega_1\Omega_2} & -\dot{\rho}_{11}\Omega_2 \end{bmatrix} \quad (18)$$

$$\mathbf{b} = \frac{1}{2} \begin{bmatrix} \Omega_1 & \rho_{12}\sqrt{\Omega_1\Omega_2} \\ \rho_{12}\sqrt{\Omega_1\Omega_2} & \Omega_2 \end{bmatrix} \quad (19)$$

$$\mathbf{c} = \frac{1}{2} \begin{bmatrix} 0 & -\dot{\rho}_{12}\sqrt{\Omega_1\Omega_2} \\ \dot{\rho}_{12}\sqrt{\Omega_1\Omega_2} & 0 \end{bmatrix} \quad (20)$$

where $\dot{\rho}_{il} = d\rho_{il}(\tau)/d\tau|_{\tau=0}$, $\ddot{\rho}_{il} = d^2\rho_{il}(\tau)/d\tau^2|_{\tau=0}$, and $\rho_{il} \triangleq \rho_{il}(0)$. Note that the diagonal elements in matrix \mathbf{c} are null, because for a stationary process, the correlation between the process and its time derivative is always null at $\tau = 0$ ($\dot{\rho}_{11} = \dot{\rho}_{22} = 0$) [15].

From [16, pp. 495–496], the conditional density of $\dot{\mathbf{Z}}$ given \mathbf{Z} is Gaussian distributed with mean matrix \mathbf{M} and covariance matrix Δ given by

$$\begin{aligned} \mathbf{M} &= \begin{bmatrix} \mathbb{E}(\dot{Z}_1|\mathbf{Z}) \\ \mathbb{E}(\dot{Z}_2|\mathbf{Z}) \end{bmatrix} \\ &= (\mathbf{c}\mathbf{b}^{-1})^* \mathbf{Z} \end{aligned} \quad (21)$$

$$\begin{aligned} \Delta &= \frac{1}{2} \begin{bmatrix} \text{Var}(\dot{Z}_1|\mathbf{Z}) & \text{Cov}(\dot{Z}_1, \dot{Z}_2|\mathbf{Z}) \\ \text{Cov}(\dot{Z}_1, \dot{Z}_2|\mathbf{Z})^* & \text{Var}(\dot{Z}_2|\mathbf{Z}) \end{bmatrix} \\ &= \mathbf{a} - \mathbf{c}\mathbf{b}^{-1}\mathbf{c}^H. \end{aligned} \quad (22)$$

Substituting (18)–(20) into (21) and (22), the conditional statistics of \dot{Z}_i given \mathbf{Z} is obtained as

$$\begin{aligned} \mathbf{M} &= \frac{1}{1-|\rho_{12}|^2} \begin{bmatrix} \rho_{12}\dot{\rho}_{12}^* Z_1 - \dot{\rho}_{12}^* \sqrt{\frac{\Omega_1}{\Omega_2}} Z_2 \\ \sqrt{\frac{\Omega_2}{\Omega_1}} \dot{\rho}_{12}^* Z_1 - \rho_{12}\dot{\rho}_{12}^* Z_2 \end{bmatrix} \\ &= \frac{1}{1-|\rho_{12}|^2} \begin{bmatrix} \rho_{12}\dot{\rho}_{12}^* R_1^{\alpha_1/2} e^{j\Theta_1} - \dot{\rho}_{12}^* \sqrt{\frac{\Omega_1}{\Omega_2}} R_2^{\alpha_2/2} e^{j\Theta_2} \\ \sqrt{\frac{\Omega_2}{\Omega_1}} \dot{\rho}_{12}^* R_1^{\alpha_1/2} e^{j\Theta_1} - \rho_{12}\dot{\rho}_{12}^* R_2^{\alpha_2/2} e^{j\Theta_2} \end{bmatrix} \end{aligned} \quad (23)$$

$$\begin{aligned} \Delta &= \frac{1}{2} \begin{bmatrix} -\Omega_1 \left(\ddot{\rho}_{11} + \frac{|\dot{\rho}_{12}|^2}{1-|\rho_{12}|^2} \right) & -\sqrt{\Omega_1 \Omega_2} \left(\ddot{\rho}_{12} + \frac{\rho_{12}^* |\dot{\rho}_{12}|^2}{1-|\rho_{12}|^2} \right) \\ -\sqrt{\Omega_1 \Omega_2} \left(\ddot{\rho}_{12}^* + \frac{\rho_{12} |\dot{\rho}_{12}^*|^2}{1-|\rho_{12}|^2} \right) & -\Omega_2 \left(\ddot{\rho}_{11} + \frac{|\dot{\rho}_{12}|^2}{1-|\rho_{12}|^2} \right) \end{bmatrix}. \end{aligned} \quad (24)$$

Finally, replacing (23) and (24) into (13)–(15), the conditional statistics of the real variates \dot{R}_i 's are obtained as

$$\begin{aligned} m_{\dot{R}_1} &= \frac{2}{\alpha_1} \frac{1}{1-|\rho_{12}|^2} \left[r_1 \text{Re} [\rho_{12}\dot{\rho}_{12}^*] - r_1^{1-\frac{\alpha_1}{2}} r_2^{\frac{\alpha_2}{2}} \right. \\ &\quad \left. \times \sqrt{\frac{\Omega_1}{\Omega_2}} \text{Re} [\dot{\rho}_{12}^* e^{j\theta_{12}}] \right] \end{aligned} \quad (25)$$

$$\begin{aligned} m_{\dot{R}_2} &= \frac{2}{\alpha_2} \frac{1}{1-|\rho_{12}|^2} \left[r_1^{\frac{\alpha_1}{2}} r_2^{1-\frac{\alpha_2}{2}} \sqrt{\frac{\Omega_2}{\Omega_1}} \text{Re} [\dot{\rho}_{12}^* e^{j\theta_{12}}] \right. \\ &\quad \left. - r_2 \text{Re} [\rho_{12}\dot{\rho}_{12}^*] \right] \end{aligned} \quad (26)$$

$$\sigma_{\dot{R}_i}^2 = -\frac{2\Omega_i r_i^{2-\alpha_i}}{\alpha_i^2} \left[\ddot{\rho}_{11} + \frac{|\dot{\rho}_{12}|^2}{1-|\rho_{12}|^2} \right], \quad i = 1, 2 \quad (27)$$

$$\sigma_{\dot{R}_1, \dot{R}_2} = -\frac{2\sqrt{\Omega_1 \Omega_2}}{\alpha_1 \alpha_2} r_1^{1-\frac{\alpha_1}{2}} r_2^{1-\frac{\alpha_2}{2}} \text{Re} \left[\left(\ddot{\rho}_{12}^* + \frac{|\dot{\rho}_{12}|^2 \rho_{12}}{1-|\rho_{12}|^2} \right) e^{j\theta_{12}} \right]. \quad (28)$$

V. NUMERICAL RESULTS AND DISCUSSIONS

The expressions obtained here for the LCR and AFD are general, and can be applied to any type of diversity (space, frequency, or time). In this section, sample numerical results are discussed assuming a space-diversity system with horizontally spaced omnidirectional antennas at the mobile station. In this case, for incoming multipath waves having equal amplitude and independent phases, the cross-correlation functions are given by [1]

$$\rho_{11}(\tau) = J_0(2\pi f_m \tau) \quad (29)$$

$$\rho_{12}(\tau) = J_0 \left(2\pi \sqrt{(f_m \tau)^2 + (d/\lambda)^2 - 2(f_m \tau)(d/\lambda \cos(\beta))} \right) \quad (30)$$

where $J_0(\cdot)$ is the zero-order Bessel function, λ is the carrier wavelength, f_m is the maximum Doppler shift in hertz, d is the antenna spacing, and $\beta \in [0, 2\pi)$ is the angle between the antenna axis and

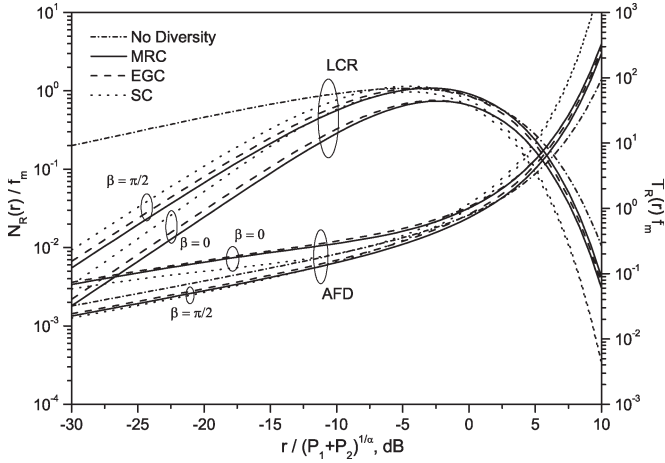


Fig. 1. LCR and AFD of SC, EGC, and MRC for $d/\lambda = 0.2$ and identical correlated fading channels ($\alpha_i = 1.5$ and $P_i = 0.5$).

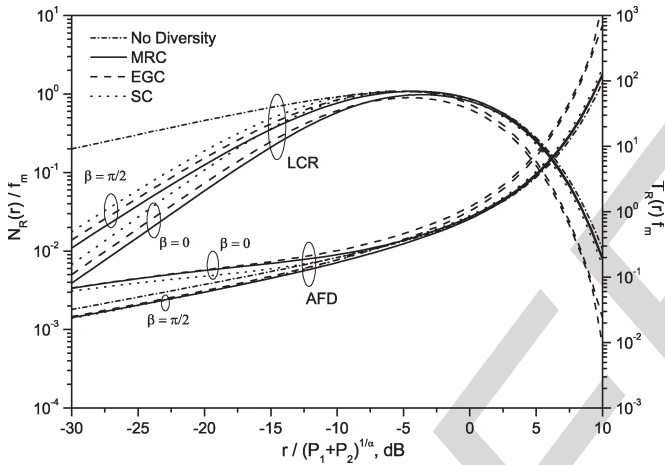


Fig. 2. LCR and AFD of SC, EGC, and MRC for $d/\lambda = 0.2$, $\alpha_i = 1.5$, and nonidentical correlated ($P_1 = 0.1$, $P_2 = 0.9$) fading channels.

the direction of the vehicle motion in radians [1]. The corresponding correlation coefficients can be calculated as

$$\rho_{11} = 1 \quad (31)$$

$$\rho_{12} = J_0(2\pi d/\lambda) \quad (32)$$

$$\dot{\rho}_{12} = 2\pi f_m \cos(\beta) J_1(2\pi d/\lambda) \quad (33)$$

$$\ddot{\rho}_{12} = (2\pi f_m)^2 \left\{ \frac{J_1(2\pi d/\lambda)}{2\pi d/\lambda} \cos(2\beta) - \cos^2(\beta) J_0(2\pi d/\lambda) \right\} \quad (34)$$

$$\ddot{\rho}_{11} = -2(\pi f_m)^2 \quad (35)$$

where $J_1(\cdot)$ is the first-order Bessel function.

Fig. 1, for the identical correlated case ($P_i = 0.5$), and Fig. 2, for the nonidentical correlated case ($P_1 = 0.1$, $P_2 = 0.9$), show the LCR (left axis) $N_R(r)/f_m$ and the AFD (right axis) $T_R(r)/f_m$ as a function of the normalized envelope $r / \sqrt[3]{P_1 + P_2}$ for SC, EGC, and MRC. The following arbitrary parameters have been used: $d/\lambda = 0.2$; $\beta = 0$; $\beta = \pi/2$; and $\alpha_i = \alpha = 1.5$. In all of them, the no diversity (ND) case has also been included. Note that for both figures, the typical performance concerning the LCR in diversity systems holds, i.e., the MRC has better performance, followed by EGC, SC, and ND. On the other hand, considering the metric AFD, the same cannot be said. For $\beta = 0$ (antenna angle parallel to the vehicle motion), the use of diversity is harmful to the performance systems.

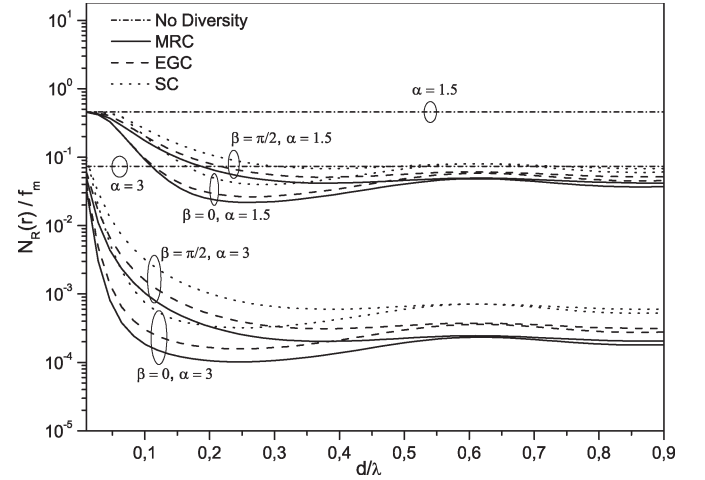


Fig. 3. LCR of SC, EGC, and MRC for $r / \sqrt[3]{P_1 + P_2} = -20$ dB and operating on identical correlated fading channels ($P_i = 0.5$) with arbitrary fading parameters.

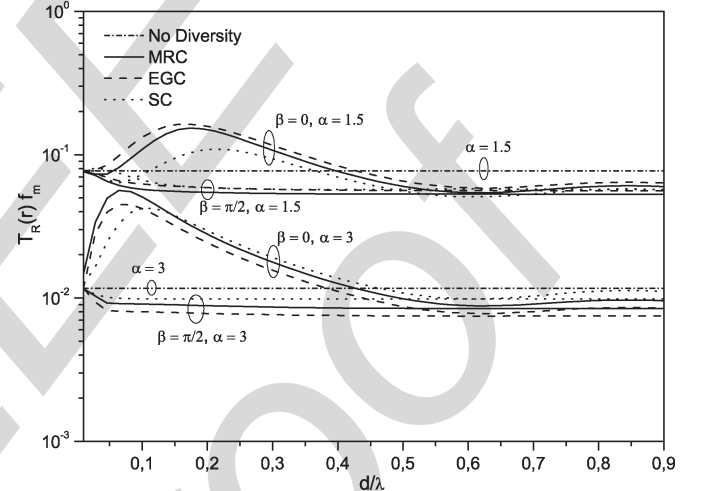


Fig. 4. AFD of SC, EGC, and MRC for $r / \sqrt[3]{P_1 + P_2} = -20$ dB and operating on identical correlated fading channels ($P_i = 0.5$) with arbitrary fading parameters.

Figs. 3 and 4, for the identical correlated case, and Figs. 5 and 6, for the nonidentical correlated case, show the LCR and AFD as a function of the parameter d/λ for SC, EGC, and MRC given a normalized envelope level at $r / \sqrt[3]{P_1 + P_2} = -20$ dB. The other parameters have been kept the same as before except for the parameter α_i , which was included in the figures ($\alpha_i = 3$). In all the cases, the curves with ND reception have also been included. From Figs. 3 and 5, it can be seen that as the antenna spacing increases, the LCR decreases, becoming oscillatory and convergent. Also, from Figs. 4 and 6, it is observed that the shape of the AFD curves for all the combining schemes is loosely dependent on the antenna spacing for $\beta = \pi/2$. Still considering the AFD metric, for $\beta = 0$, depending on the antenna spacing, the use of diversity can be advantageous or not for the performance system. It is evident from Figs. 3–6 that an enhancement of the fading conditions (increasing α) implies an improvement of performance.

Fig. 7 compares our exact analytical expressions with some simulation results. The curves were plotted for identical correlated fading channels with $\alpha_i = \alpha = 3$, $\beta = 0$, and $d/\lambda = 0.2$. Note the very good agreement between the curves. For the AFD metric, due to the proximity between the EGC and MRC curves, only for the former simulation data were presented. A myriad of other cases has been

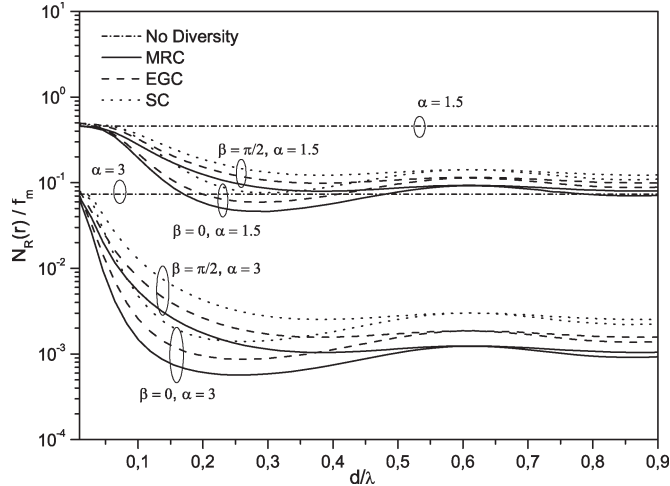


Fig. 5. LCR of SC, EGC, and MRC for $r/\sqrt[3]{P_1 + P_2} = -20$ dB and operating on nonidentical correlated fading channels ($P_1 = 0.1, P_2 = 0.9$) with arbitrary fading parameters.

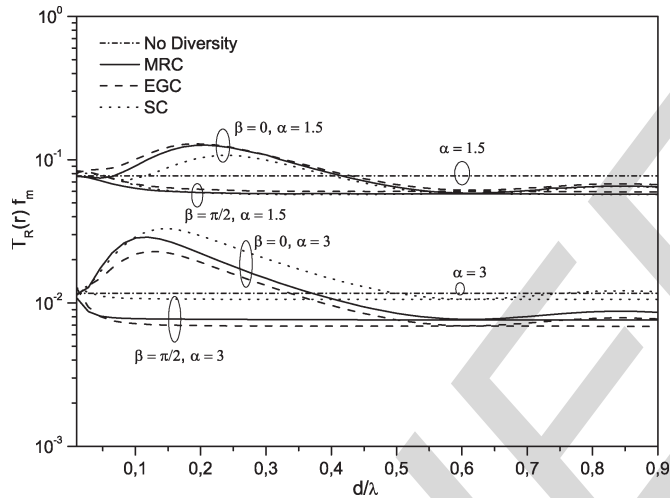


Fig. 6. AFD of SC, EGC, and MRC for $r/\sqrt[3]{P_1 + P_2} = -20$ dB and operating on nonidentical correlated fading channels ($P_1 = 0.1, P_2 = 0.9$) with arbitrary fading parameters.

exhaustively investigated, and a very good match has been observed in all of them.

VI. CONCLUSION

Exact formulas for the LCR and AFD of dual-branch SC, EGC, and MRC techniques operating on nonidentical correlated Weibull fading environment have been presented. The expressions obtained here are general and can be applied to any type of diversity (space, frequency, or time). Furthermore, some numerical results were discussed by specializing the general expressions to a space-diversity system using horizontally spaced antennas at a mobile station. Interestingly, situations are found for which the use of diversity may be deleterious. Some simulation results were presented, and a very good adjustment between the analytical and simulated curves was observed.

APPENDIX A

The goal of this Appendix is to obtain each term of the complex covariance matrix Φ given in (16). By expanding (16), the matrices \mathbf{a} ,

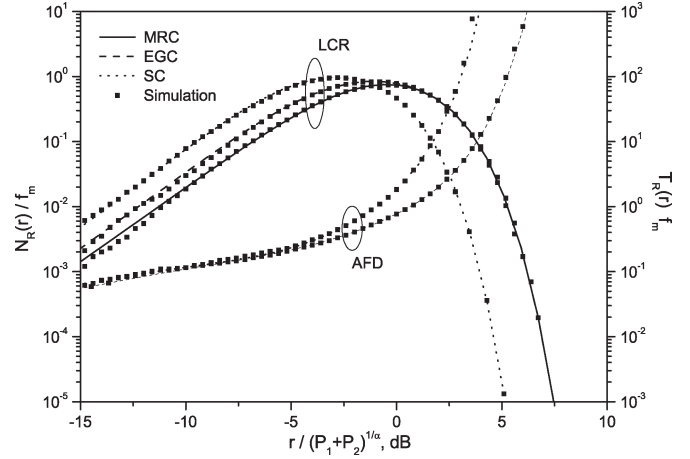


Fig. 7. Comparison between analytical and simulated curves of the LCR and AFD for SC, EGC, and MRC operating on identical correlated fading channels ($P_i = 0.5$), $d/\lambda = 0.2$, $\alpha_i = 3$, and $\beta = 0$.

b, and **c** can be formulated as

$$\mathbf{a} = \frac{1}{2} \begin{bmatrix} \text{Cov}(\dot{Z}_1, \dot{Z}_1) & \text{Cov}(\dot{Z}_1, \dot{Z}_2) \\ \text{Cov}(\dot{Z}_2, \dot{Z}_1) & \text{Cov}(\dot{Z}_2, \dot{Z}_2) \end{bmatrix} \quad (36)$$

$$\mathbf{b} = \frac{1}{2} \begin{bmatrix} \text{Cov}(Z_1, Z_1) & \text{Cov}(Z_1, Z_2) \\ \text{Cov}(Z_2, Z_1)^* & \text{Cov}(Z_2, Z_2) \end{bmatrix} \quad (37)$$

$$\mathbf{c} = \frac{1}{2} \begin{bmatrix} \text{Cov}(\dot{Z}_1, Z_1) & \text{Cov}(\dot{Z}_1, Z_2) \\ \text{Cov}(\dot{Z}_2, Z_1) & \text{Cov}(\dot{Z}_2, Z_2) \end{bmatrix}. \quad (38)$$

Defining $\rho_{il}(\tau)$ as in (17), and noting that $\dot{\rho}_{il} = d\rho_{il}(\tau)/d\tau|_{\tau=0}$, $\ddot{\rho}_{il} = d^2\rho_{il}(\tau)/d\tau^2|_{\tau=0}$, and $\rho_{il} \triangleq \rho_{il}(0)$, it follows that

$$\begin{aligned} \text{Cov}(Z_1(t), Z_1(t+\tau)) &= \rho_{11}(\tau)\Omega_1 \\ \therefore \text{Cov}(\dot{Z}_1, \dot{Z}_1) &= -\Omega_1 \left. \frac{d^2\rho_{11}(\tau)}{d\tau^2} \right|_{\tau=0} \\ &= -\ddot{\rho}_{11}\Omega_1 \end{aligned} \quad (39)$$

$$\begin{aligned} \text{Cov}(Z_1(t), Z_2(t+\tau)) &= \rho_{12}(\tau)\sqrt{\Omega_1\Omega_2} \\ \therefore \text{Cov}(\dot{Z}_1, \dot{Z}_2) &= -\sqrt{\Omega_1\Omega_2} \left. \frac{d^2\rho_{12}(\tau)}{d\tau^2} \right|_{\tau=0} \\ &= -\ddot{\rho}_{12}\sqrt{\Omega_1\Omega_2} \end{aligned} \quad (40)$$

As $\rho_{12} = \rho_{21}^*$, then $\text{Cov}(\dot{Z}_1, \dot{Z}_2) = \text{Cov}(\dot{Z}_2, \dot{Z}_1)$

$$\begin{aligned} \text{Cov}(Z_2(t), Z_2(t+\tau)) &= \rho_{22}(\tau)\Omega_2 \\ \therefore \text{Cov}(\dot{Z}_2, \dot{Z}_2) &= -\Omega_2 \left. \frac{d^2\rho_{22}(\tau)}{d\tau^2} \right|_{\tau=0} \\ &= -\ddot{\rho}_{22}\Omega_2 \\ &= -\ddot{\rho}_{11}\Omega_2. \end{aligned} \quad (41)$$

Knowing that $\rho_{11}(\tau) = \rho_{22}(\tau) = J_0(2\pi f_m \tau)$, then $\ddot{\rho}_{11} = \ddot{\rho}_{22}$

$$\text{Cov}(Z_i, Z_i) = \Omega_i \rho_{ii}(\tau)|_{\tau=0} = \Omega_i \quad (42)$$

$$\text{Cov}(\dot{Z}_1, Z_1) = \Omega_1 \left. \frac{d\rho_{11}(\tau)}{d\tau} \right|_{\tau=0} = 0 = \text{Cov}(\dot{Z}_2, Z_2) \quad (43)$$

since $\rho_{11}(\tau) = \rho_{22}(\tau) = J_0(2\pi f_m \tau)$

$$\begin{aligned} \text{Cov}(Z_1, Z_2) &= \sqrt{\Omega_1\Omega_2} \rho_{12}(\tau)|_{\tau=0} \\ &= \rho_{12} \sqrt{\Omega_1\Omega_2} \end{aligned} \quad (44)$$

$$\begin{aligned} \text{Cov}(\dot{Z}_1, Z_2) &= -\sqrt{\Omega_1\Omega_2} \left. \frac{d\rho_{12}(\tau)}{d\tau} \right|_{\tau=0} \\ &= -\dot{\rho}_{12}\sqrt{\Omega_1\Omega_2} \end{aligned} \quad (45)$$

$$\begin{aligned} \text{Cov}(\dot{Z}_2, Z_1) &= \sqrt{\Omega_1\Omega_2} \left. \frac{d\rho_{12}(\tau)}{d\tau} \right|_{\tau=0} \\ &= \dot{\rho}_{12}\sqrt{\Omega_1\Omega_2}. \end{aligned} \quad (46)$$

APPENDIX B

In this Appendix, we derive the relations given in (13)–(15).

- Calculation of $E(\dot{R}_i|\mathbf{Z})$.

Differentiating both sides of (1) with respect to time yields

$$\dot{Z}_i = \frac{\alpha_i}{2} R_i^{\alpha_i/2-1} \dot{R}_i e^{j\Theta_i} + j\dot{\Theta}_i R_i^{\alpha_i/2} e^{j\Theta_i}. \quad (47)$$

Rearranging the terms and applying the mean operator gives

$$\begin{aligned} \dot{R}_i &= \frac{2}{\alpha_i} R_i^{1-\alpha_i/2} \text{Re} [\dot{Z}_i e^{-j\Theta_i}] \\ \therefore E(\dot{R}_i|\mathbf{Z}) &= \frac{2}{\alpha_i} R_i^{1-\alpha_i/2} \text{Re} [E(\dot{Z}_i|\mathbf{Z}) e^{-j\Theta_i}]. \end{aligned} \quad (48)$$

- Calculation of $\text{Var}(\dot{R}_i|\mathbf{Z})$.

Using the identity $\text{Re}[x] = (x + x^*)/2$, we can write

$$\begin{aligned} \dot{R}_i &= \frac{2}{\alpha_i} R_i^{1-\alpha_i/2} \text{Re} [\dot{Z}_i e^{-j\Theta_i}] \\ &= \frac{2}{\alpha_i} R_i^{1-\alpha_i/2} \left(\frac{\dot{Z}_i e^{-j\Theta_i} + \dot{Z}_i^* e^{j\Theta_i}}{2} \right). \end{aligned} \quad (49)$$

Using the definition of complex variance, it follows that

$$\text{Var}(\dot{R}_i|\mathbf{Z}) = E(|\dot{R}_i|^2|\mathbf{Z}) - |E(\dot{R}_i|\mathbf{Z})|^2. \quad (50)$$

The first term of the right side of the equality given in (50) can be expressed as

$$\begin{aligned} E(|\dot{R}_i|^2|\mathbf{Z}) &= E(\dot{R}_i \dot{R}_i^*|\mathbf{Z}) \\ &= E\left(\left(\frac{2}{\alpha_i} R_i^{1-\alpha_i/2} \frac{\dot{Z}_i e^{-j\Theta_i} + \dot{Z}_i^* e^{j\Theta_i}}{2}\right)\right. \\ &\quad \left.\times \left(\frac{2}{\alpha_i} R_i^{1-\alpha_i/2} \frac{\dot{Z}_i^* e^{j\Theta_i} + \dot{Z}_i e^{-j\Theta_i}}{2}\right) \middle| \mathbf{Z}\right) \\ &= \frac{R_i^{2-\alpha_i}}{\alpha_i^2} \left(2E(|\dot{Z}_i|^2|\mathbf{Z}) + e^{-j2\Theta_i} E(\dot{Z}_i^2|\mathbf{Z}) \right. \\ &\quad \left. + e^{j2\Theta_i} E((\dot{Z}_i^*)^2|\mathbf{Z}) \right). \end{aligned} \quad (51)$$

The second term of the right side of the equality given in (50) can be expressed as

$$\begin{aligned} |E(\dot{R}_i|\mathbf{Z})|^2 &= E(\dot{R}_i|\mathbf{Z}) E(\dot{R}_i^*|\mathbf{Z}) \\ &= E\left(\frac{2}{\alpha_i} R_i^{1-\alpha_i/2} \frac{\dot{Z}_i e^{-j\Theta_i} + \dot{Z}_i^* e^{j\Theta_i}}{2} \middle| \mathbf{Z}\right) \\ &\quad \times E\left(\frac{2}{\alpha_i} R_i^{1-\alpha_i/2} \frac{\dot{Z}_i^* e^{j\Theta_i} + \dot{Z}_i e^{-j\Theta_i}}{2} \middle| \mathbf{Z}\right) \end{aligned}$$

$$\begin{aligned} &= \frac{R_i^{2-\alpha_i}}{\alpha_i^2} \left(2|E(\dot{Z}_i|\mathbf{Z})|^2 + e^{-j2\Theta_i} E(\dot{Z}_i|\mathbf{Z})^2 \right. \\ &\quad \left. + e^{j2\Theta_i} E(\dot{Z}_i^*|\mathbf{Z})^2 \right). \end{aligned} \quad (52)$$

By substituting (51) and (52) into (50), it follows that

$$\begin{aligned} \text{Var}(\dot{R}_i|\mathbf{Z}) &= \frac{2R_i^{2-\alpha_i}}{\alpha_i^2} \left(\text{Var}(\dot{Z}_i|\mathbf{Z}) \right. \\ &\quad \left. + \text{Re} [e^{-j2\Theta_i} \text{Cov}(\dot{Z}_i, \dot{Z}_i^*|\mathbf{Z})^*] \right). \end{aligned} \quad (53)$$

The next step is to prove that the term $\text{Cov}(\dot{Z}_i, \dot{Z}_i^*|\mathbf{Z})^*$ is zero. Following a similar procedure to that applied in Section IV, it can be verified that such term is always zero. Then, (53) reduces to

$$\text{Var}(\dot{R}_i|\mathbf{Z}) = \frac{2R_i^{2-\alpha_i}}{\alpha_i^2} \text{Var}(\dot{Z}_i|\mathbf{Z}). \quad (54)$$

- Calculation of $\text{Cov}(\dot{R}_i, \dot{R}_l|\mathbf{Z})$.

The calculation of $\text{Cov}(\dot{R}_i, \dot{R}_l|\mathbf{Z})$ follows a similar rationale. For brevity, it will not be presented here.

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