Crossing Rates and Fade Durations for Diversity-Combining Schemes over α-μ Fading Channels

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***Abstract*— This paper derives exact expressions for the level crossing rate and average fade duration of multibranch pure-selection, equal-gain, and maximal-ratio combiners operating over independent non-identical α-μ (also called generalized Gamma or Stacy) fading channels. The derived expressions are in closed form for pure-selection combining and in integral form for equal-gain and maximal-ratio combining. For the two latter schemes, accurate closed-form approximations are then provided. The analytical results are validated by reducing the general expressions to known particular cases and, more generally, by means of simulation. Numerical examples are also given to illustrate the high accuracy of the proposed closed-form approximations.**

***Index Terms*— Approximation methods, average fade duration, diversity methods, generalized fading channels, level crossing rate.**

I. Introduction

The α-μ fading model was recently proposed in [1], [2] by considering two important phenomena inherent to radio propagation, namely non-linearity and clustering. The model couples mathematical ease with an enormous flexibility, providing a very good fit to measured data over a wide range of fading conditions. The α-μ distribution is written in terms of physically-based fading parameters, namely α and μ, which describe the non-linearity of the propagation medium and the multipath wave clustering, respectively. This distribution is also known as generalized Gamma or Stacy distribution [1], [2].

In wireless systems, the envelope of the received signal varies randomly because of multipath fading, and diversity-combining techniques are often utilized to counteract this random signal variation. The level crossing rate (LCR) and average fade duration (AFD) are important metrics for evaluating the dynamic performance of diversity systems. For this reason, these metrics have been widely investigated in the literature for the well-known fading channels [3]–[6]. This work generalizes previous ones by providing exact expressions for LCR and AFD of multibranch pure-selection combining (PSC), equal-gain combining (EGC), and maximal-ratio combining (MRC) schemes operating over independent non-identical α-μ fading channels. For PSC, the exact expressions are obtained in closed form. On the other hand, for EGC and MRC, they are given in multifold integral form. Alternatively, highly accurate simple closed-form approximations are also provided for the EGC and MRC schemes. The analytical results derived here are validated by reducing the general expressions to known particular cases and, more generally, by means of simulation. Numerical examples are also given to illustrate the high accuracy of the proposed closed-form approximations.

II. The α-μ Fading Model

In this Section, we revisit the α-μ fading model proposed in [1], [2]. The α-μ propagation condition considers a signal composed of clusters of multipath waves traveling in a non-homogeneous environment. The resulting envelope is obtained as a non-linear function of the modulus of the sum of the multipath components. Such a non-linearity is manifested in terms of a power parameter, so that the resulting signal intensity is obtained not simply as the modulus of the sum of the multipath components, but as this modulus to a certain given exponent.

Assuming that the received signal at the ith branch ($i = 1, ..., M$) includes a certain number n_i of multipath clusters, the resulting α-μ envelope R_i at the ith branch is written as [1], [2]

$$R_i^{\alpha_i} = \sum_{l=1}^{n_i} \left(X_{il}^2 + Y_{il}^2 \right), \tag{1}$$

where $\alpha_i > 0$ is the power parameter, X_{il} and Y_{il} are zero-mean mutually independent Gaussian processes with identical variances $V(X_{il}) = V(Y_{il}) = \sigma_i^2$, with $V(\cdot)$ denoting variance. The corresponding α-μ probability density function (PDF) $f_{R_i}(\cdot)$ of R_i, as given in [1], [2], is

$$f_{R_i}(r_i) = \frac{\alpha_i \, \mu_i^{\mu_i} r_i^{\alpha_i \mu_i - 1}}{\hat{r_i}^{\alpha_i \mu_i} \Gamma(\mu_i)} \exp\left(-\mu_i \frac{r_i^{\alpha_i}}{\hat{r_i}^{\alpha_i}} \right), \tag{2}$$

where $\hat{r_i} = \sqrt[\alpha_i]{E(R_i^{\alpha_i})} = \sqrt[\alpha_i]{2\mu_i \sigma_i^2}$ is the α_i-root mean value of $R_i^{\alpha_i}$, $\Gamma(z) = \int_0^\infty t^{z-1} \exp(-t) dt$ is the gamma function, $E(\cdot)$ denotes expectation, and $\mu_i > 0$ is a real extension of the parameter n_i, given by the inverse of the normalized variance of $R_i^{\alpha_i}$, i.e, $\mu_i = E^2(R_i^{\alpha_i})/V(R_i^{\alpha_i})$. For $\mu_i = 1$, (2) reduces to the Weibull PDF, whereas for $\alpha_i = 2$ it reduces to the Nakagami-m PDF. From (2), the kth moment $E(R_i^k)$ can be

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obtained as

$$E(R_i^k) = \hat{r_i}^k \frac{\Gamma(\mu_i + k/\alpha_i)}{\mu_i^{k/\alpha_i}\Gamma(\mu_i)}. \quad (3)$$

The cumulative distribution function (CDF) $F_{R_i}(\cdot)$ of R_i is given by [1], [2]

$$F_{R_i}(r_i) = \frac{\Gamma(\mu_i, \mu_i r_i^{\alpha_i}/\hat{r_i}^{\alpha_i})}{\Gamma(\mu_i)}, \quad (4)$$

where $\Gamma(z, y) = \int_0^y t^{z-1}\exp(-t)dt$ is the incomplete gamma function.

For isotropic scattering, the time derivatives $\dot{X}_{il}$ and $\dot{Y}_{il}$ of X_{il} and Y_{il}, respectively, are zero-mean Gaussian variates with variances $\dot{\sigma_i}^2 = 2\pi^2 f_m^2 \sigma_i^2$ [7], where f_m is the maximum Doppler shift in Hz. Correspondingly, from (1), the time derivative $\dot{R}_i$ of R_i, given R_i, is zero-mean Gaussian distributed with variance $\sigma^2_{\dot{R}_i} = \frac{r_i^{2-\alpha_i}}{\alpha_i^2 \mu_i}\Omega_i 4\pi^2 f_m^2$, in which $\Omega_i = \hat{r_i}^{\alpha_i}$. These results have been derived and used in [2] to obtain the LCR $N_{R_i}(r)$ and AFD $T_{R_i}(r)$ of the α-μ envelope R_i, yielding [2]

$$N_{R_i}(r) = \frac{\sqrt{2\pi}\, f_m\, r^{\alpha_i(\mu_i-0.5)}\, \mu_i^{\mu_i-0.5}}{\Gamma(\mu_i)\,\Omega_i^{\mu_i-0.5}} \exp\left(-\frac{\mu_i\, r^{\alpha_i}}{\Omega_i}\right), \quad (5)$$

$$T_{R_i}(r) = \frac{\Gamma(\mu_i, \mu_i\, r^{\alpha_i}/\Omega_i)\,\Omega_i^{\mu_i-0.5}}{\sqrt{2\pi}\, f_m\, r^{\alpha_i(\mu_i-0.5)}\mu_i^{\mu_i-0.5}} \exp\left(\frac{\mu_i\, r^{\alpha_i}}{\Omega_i}\right). \quad (6)$$

III. LCR AND AFD

The LCR is defined as the average number signal crossings per second at a given level, in the negative or positive direction. Denoting the time derivative of the envelope R as $\dot{R}$ and the crossing level as r, the LCR is estimated as [6]

$$N_R(r) = \int_0^\infty \dot{r} f_{R,\dot{R}}(r, \dot{r})d\dot{r}, \quad (7)$$

where $f_{R,\dot{R}}(\cdot,\cdot)$ is the joint PDF of R and $\dot{R}$. The AFD is defined as the mean time the received envelope remains below a given threshold r after crossing it in the negative direction, given by

$$T_R(r) = \frac{F_R(r)}{N_R(r)}, \quad (8)$$

where $F_R(\cdot)$ is the CDF of R. In the following, R and $\dot{R}$ denote the combiner output envelope and its time derivative, respectively.

A. Pure-Selection Combining

In PSC, the received signals are continuously monitored so that the best signal is selected. Thus, the combiner output envelope R can be written as

$$R = \max_{i=1,\ldots,M}\{R_i\}. \quad (9)$$

In [8], a general formulation for PSC systems operating over independent fading channels has been obtained as

$$N_R(r) = \sum_{i=1}^M N_{R_i}(r) \prod_{\substack{j=1\\ j\neq i}}^M F_{R_j}(r). \quad (10)$$

In our case, $F_{R_j}(\cdot)$ and $N_{R_i}(\cdot)$ are given by (4) and (5), respectively. In the same way, since for independent branches $F_R(r) = \prod_{i=1}^M F_{R_i}(r)$, by replacing this and (10) into (8), and after some algebraic manipulations, it can be shown that

$$T_R^{-1}(r) = \sum_{i=1}^M T_{R_i}^{-1}(r), \quad (11)$$

where $T_{R_i}(r)$ is given by (6). It is noteworthy that (11) is indeed general and applies to any independent-fading scenario. To the best of the authors' knowledge, (11) is new.

B. Equal-Gain Combining

In EGC, the received signals with envelopes R_i are cophased and added so that the combiner output envelope R, already taking into account the resultant noise power at the combiner output, is written as

$$R = \frac{1}{\sqrt{M}}\sum_{i=1}^M R_i. \quad (12)$$

Differentiating both sides of (12), we have

$$\dot{R} = \frac{1}{\sqrt{M}}\sum_{i=1}^M \dot{R}_i. \quad (13)$$

In [3], it has been shown that $f_{R,\dot{R}}(\cdot,\cdot)$ can be written in terms of the joint PDF $f_{R_1,R_2,\ldots,R_M,\dot{R}}(r_1, r_2, \ldots, r_M, \dot{r})$ of $R_1, \ldots, R_M$ and $\dot{R}$ as

$$f_{R,\dot{R}}(r,\dot{r}) = \sqrt{M}\overbrace{\int_0^{\sqrt{M}r}\int_0^{\sqrt{M}r-r_M}\cdots\int_0^{\sqrt{M}r-\sum_{i=3}^M r_i}}^{M-1} f_{R_1,R_2,\ldots,R_M,\dot{R}}\left(\left(\sqrt{M}r - \sum_{i=2}^M r_i\right), r_2, \ldots, r_M, \dot{r}\right) dr_2 \ldots dr_M. \quad (14)$$

Of course,

$$f_{R_1,R_2,\ldots,R_M,\dot{R}}(r_1, r_2, \ldots, r_M, \dot{r}) = f_{\dot{R}|R_1,R_2,\ldots,R_M,}(\dot{r}|r_1, r_2, \ldots, r_M) \times f_{R_1,\ldots,R_M}(r_1, \ldots, r_M). \quad (15)$$

Because the branches are assumed to be independent, then

$$f_{R_1,\ldots,R_M}(r_1, \ldots, r_M) = \prod_{i=1}^M f_{R_i}(r_i), \quad (16)$$

where each $f_{R_i}(\cdot)$ is given by (2). On the other hand, from (13), we note that

$$f_{\dot{R}|R_1,R_2,\ldots,R_M,}(\dot{r}|r_1, r_2, \ldots, r_M) \sim N\left(0, \sum_{i=1}^M \sigma^2_{\dot{R}_i}/M\right), \quad (17)$$

with $\sigma^2_{\dot{R}_i} = \frac{r_i^{2-\alpha_i}}{\alpha_i^2 \mu_i}\Omega_i 4\pi^2 f_m^2$ and where $N(a, b)$ denotes an $a-$mean and $b-$variance Gaussian distribution. Now, with (15), (16), and (17) into (14), and then this into (7), and after algebraic manipulations, it follows that

$$N_R(r) = \sqrt{2\pi} f_m \overbrace{\int_0^{\sqrt{M}r} \int_0^{\sqrt{M}r - r_M} \cdots \int_0^{\sqrt{M}r - \sum_{i=3}^{M} r_i}}^{M-1}$$
$$\sqrt{\frac{\left(\sqrt{M}r - \sum_{i=2}^{M} r_i\right)^{2-\alpha_1} \Omega_1}{\alpha_1^2 \mu_1} + \sum_{i=2}^{M} \frac{r_i^{2-\alpha_i} \Omega_i}{\alpha_i^2 \mu_i}}$$
$$\times f_{R_1}\left(\sqrt{M}r - \sum_{i=2}^{M} r_i\right) \prod_{i=2}^{M} f_{R_i}(r_i) dr_2 \ldots dr_{M-1} dr_M. \quad (18)$$

It has been also shown in [3] that

$$F_R(r) = \int_0^{\sqrt{M}r} \int_0^{\sqrt{M}r - r_M} \cdots \int_0^{\sqrt{M}r - \sum_{i=2}^{M} r_i}$$
$$\prod_{i=1}^{M} f_{R_i}(r_i) dr_1 dr_2 \ldots dr_M. \quad (19)$$

Replacing (18) and (19) into (8), the AFD is obtained.

C. Maximal-Ratio Combining

In MRC, the received signals are cophased, each signal is amplified appropriately for optimal combining, and the resultant signals are added so that the combiner output envelope R is given by

$$R = \sqrt{\sum_{i=1}^{M} R_i^2}. \quad (20)$$

The time derivative $\dot{R}$ of R can be expressed as

$$\dot{R} = \sum_{i=1}^{M} \frac{R_i}{R} \dot{R}_i. \quad (21)$$

The MRC analysis follows the same rationale as for the EGC one, but now, from [3],

$$f_{R,\dot{R}}(r,\dot{r}) = \overbrace{\int_0^{r} \int_0^{\sqrt{r^2 - r_M^2}} \cdots \int_0^{\sqrt{r^2 - \sum_{i=3}^{M} r_i^2}}}^{M-1} \frac{r}{\sqrt{r^2 - \sum_{i=2}^{M} r_i^2}}$$
$$\times f_{R_1,\ldots,R_M,\dot{R}}\left(\left(\sqrt{r^2 - \sum_{i=2}^{M} r_i^2}\right), \ldots, r_M, \dot{r}\right) dr_2 \ldots dr_M, \quad (22)$$

$$F_R(r) = \int_0^{r} \int_0^{\sqrt{r^2 - r_M^2}} \cdots \int_0^{\sqrt{r^2 - \sum_{i=3}^{M} r_i^2}} \int_0^{\sqrt{r^2 - \sum_{i=2}^{M} r_i^2}}$$
$$\prod_{i=1}^{M} f_{R_i}(r_i) dr_1 dr_2 \ldots dr_{M-1} dr_M, \quad (23)$$

and, from (21),

$$f_{\dot{R}|R_1,R_2,\ldots,R_M,}(\dot{r}|r_1, r_2, \ldots, r_M) \sim N\left(0, \sum_{i=1}^{M} r_i^2 \sigma_{\dot{R}_i}^2 / r^2\right), \quad (24)$$

so that, with the use of (15) and (16), it follows that

$$N_R(r) = \sqrt{2\pi} f_m \overbrace{\int_0^{r} \int_0^{\sqrt{r^2 - r_M^2}} \cdots \int_0^{\sqrt{r^2 - \sum_{i=3}^{M} r_i^2}}}^{M-1}$$
$$\frac{1}{\sqrt{r^2 - \sum_{i=2}^{M} r_i^2}} \sqrt{\frac{\left(r^2 - \sum_{i=2}^{M} r_i^2\right)^{\frac{4-\alpha_1}{2}} \Omega_1}{\alpha_1^2 \mu_1} + \sum_{i=2}^{M} \frac{r_i^{4-\alpha_i} \Omega_i}{\alpha_i^2 \mu_i}}$$
$$\times f_{R_1}\left(\sqrt{r^2 - \sum_{i=2}^{M} r_i^2}\right) \prod_{i=2}^{M} f_{R_i}(r_i) dr_2 \ldots dr_{M-1} dr_M. \quad (25)$$

Replacing (25) and (23) into (8), the AFD is obtained.

D. Special Cases

As a check, we compare our general EGC and MRC expressions with known particular-case solutions. By setting $\mu_i = 1$ (Weibull case), our results reduce to those of [3]. In the same way, by setting $\alpha_i = 2$ (Nakagami-*m* case), our formulations reduce to those obtained in [5, Eqs. 23-24] (balanced EGC), [5, Eqs. 36-37] (balanced MRC), [9, Eqs. 29-30] (unbalanced MRC), and [9, Eqs. 39-40] (unbalanced EGC). In addition, for MRC with $M = 2$, $\mu = 1$, and $\alpha = 4$, our formulations reduce to those of [3, Eqs. 16-17].

IV. Closed-Form Approximations

The formulations developed in the previous Sections are general and exact. On the other hand, apart from those very special cases for which closed-form expressions are found, the solution by means of multifold integrals may not be attractive, as far as computational evaluation is concerned. It is certainly of interest to find some accurate approximations that can be used in order to circumvent the application of multifold integrals. In [10], it has been shown that the sum of Weibull variates can be closely approximated by an α-μ variate. Here, we extend the idea presented in [10] by approximating the sum of α-μ *random processes* by another α-μ *random process*, in order to derive simple closed-form approximations for the LCR and AFD of EGC and MRC systems over α-μ fading channels. The motivation for this comes from the fact that the sum of α-μ powers is also α-μ distributed. More specifically, assuming that Z_i, $i = 1, ..., M$ are i.i.d. α-μ variates, with parameters α, μ, and $\hat{r}$, then, from the proposed model, $Z^\alpha = \sum_{i=1}^{M} Z_i^\alpha$ is also α-μ distributed with parameters α, μM, and $\sqrt[\alpha]{\hat{r}^\alpha M}$. As shall be seen, the resulting approximations prove highly accurate. The derivations are detailed next.

From the above, our proposal is to approximate the exact LCR and AFD of EGC and MRC derived in the previous Section by the LCR and AFD of a properly-chosen α-μ variate, i.e. [1], [2]

$$N_R(r) \approx \frac{\sqrt{2\pi} f_m r^{\alpha(\mu-0.5)} \mu^{\mu-0.5}}{\Gamma(\mu) \Omega^{\mu-0.5}} \exp\left(-\frac{\mu r^\alpha}{\Omega}\right), \quad (26)$$

$$T_R(r) \approx \frac{\Gamma(\mu, \mu r^\alpha / \Omega) \Omega^{\mu-0.5}}{\sqrt{2\pi} f_m r^{\alpha(\mu-0.5)} \mu^{\mu-0.5}} \exp\left(\frac{\mu r^\alpha}{\Omega}\right), \quad (27)$$

where the parameters of the approximate formulas, namely α, μ, and Ω must be evaluated for the appropriate application.

Note that (26) and (27) have the same functional form as of (5) and (6), respectively. Now, in order to render (26) and (27) good approximations, we shall use moment-based estimators for α, μ, and Ω to calculate these parameters from the exact moments of the EGC and MRC combiner output R as follows.

A. Equal-Gain Combining

Assume, for the present, the knowledge of $E(R)$, $E(R^2)$, and $E(R^4)$. Then, moment-based estimators for α, μ, and Ω can be written based on [1], [2] as

$$\frac{\Gamma^2\left(\mu+\frac{1}{\alpha}\right)}{\Gamma(\mu)\,\Gamma\left(\mu+\frac{2}{\alpha}\right)-\Gamma^2\left(\mu+\frac{1}{\alpha}\right)}=\frac{E^2(R)}{E(R^2)-E^2(R)}, \tag{28}$$

$$\frac{\Gamma^2\left(\mu+\frac{2}{\alpha}\right)}{\Gamma(\mu)\,\Gamma\left(\mu+\frac{4}{\alpha}\right)-\Gamma^2\left(\mu+\frac{2}{\alpha}\right)}=\frac{E^2(R^2)}{E(R^4)-E^2(R^2)}, \tag{29}$$

$$\Omega=\left[\frac{\mu^{\frac{1}{\alpha}}\,\Gamma(\mu)E(R)}{\sqrt{M}\,\Gamma\left(\mu+\frac{1}{\alpha}\right)}\right]^{\alpha}. \tag{30}$$

The system of transcendental equations (28) and (29) must be numerically solved for α and μ. Most popular computing softwares have built-in routines for accomplishing this task in an efficient and straightforward manner. In *MATHEMATICA*, for instance, the function FindRoot can be used for this. Having obtained α and μ, Ω is then estimated as in (30). It remains to find the exact moments $E(R)$, $E(R^2)$, and $E(R^4)$ required into (28), (29), and (30), which were assumed to be known. By using the multinomial expansion, these moments are obtained from (12) in terms of the moments of the α-μ summands as [10]

$$E(R^n)=\sum_{n_1=0}^{n}\sum_{n_2=0}^{n_1}\cdots\sum_{n_{M-1}=0}^{n_{M-2}}\binom{n}{n_1}\binom{n_1}{n_2}\cdots\binom{n_{M-2}}{n_{M-1}} E(R_1^{n-n_1})E(R_2^{n_1-n_2})\ldots E(R_M^{n_{M-1}}), \tag{31}$$

where the required α-μ moments are given in (3).

B. Maximal-Ratio Combining

For MRC, assume, for the present, the knowledge of $E(R^2)$, $E(R^4)$, and $E(R^8)$. Accordingly, appropriate moment-based estimators for α, μ, and Ω can be written as [1], [2]

$$\frac{\Gamma^2\left(\mu+\frac{2}{\alpha}\right)}{\Gamma(\mu)\,\Gamma\left(\mu+\frac{4}{\alpha}\right)-\Gamma^2\left(\mu+\frac{2}{\alpha}\right)}=\frac{E^2(R^2)}{E(R^4)-E^2(R^2)}, \tag{32}$$

$$\frac{\Gamma^2\left(\mu+\frac{4}{\alpha}\right)}{\Gamma(\mu)\,\Gamma\left(\mu+\frac{8}{\alpha}\right)-\Gamma^2\left(\mu+\frac{4}{\alpha}\right)}=\frac{E^2(R^4)}{E(R^8)-E^2(R^4)}, \tag{33}$$

$$\Omega=\left[\frac{\mu^{\frac{2}{\alpha}}\,\Gamma(\mu)E(R^2)}{\Gamma\left(\mu+\frac{2}{\alpha}\right)}\right]^{\frac{\alpha}{2}}. \tag{34}$$

As before, by multinomial expansion, the exact moments $E(R^2)$, $E(R^4)$, and $E(R^8)$ required into (32), (33), and (34)

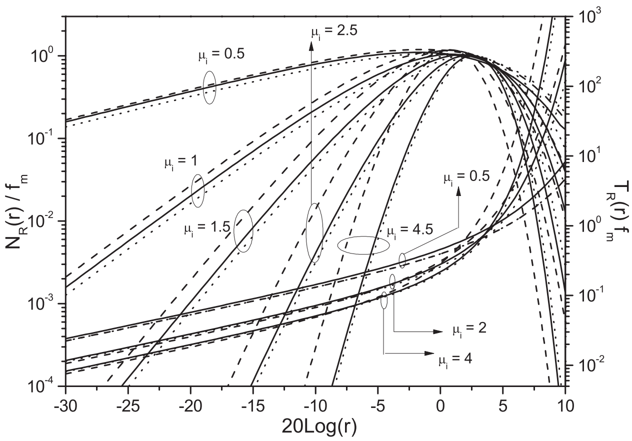

Fig. 1. LCR and AFD of PSC, EGC, and MRC techniques for α-μ fading channels (solid lines → EGC, dotted lines → MRC, dashed lines → PSC, $M = 2$, $\Omega_i = 1$, $\alpha_i = 1.5$, and varying μ_i).

can be evaluated from (20) as

$$E(R^{2n})=\sum_{n_1=0}^{n}\sum_{n_2=0}^{n_1}\cdots\sum_{n_{M-1}=0}^{n_{M-2}}\binom{n}{n_1}\binom{n_1}{n_2}\cdots\binom{n_{M-2}}{n_{M-1}} E(R_1^{2(n-n_1)})E(R_2^{2(n_1-n_2)})\ldots E(R_M^{2(n_{M-1})}). \tag{35}$$

It is noteworthy that the above closed-form approximations are very simple. Moreover, the determination of the required parameters is straightforward. Not less importantly, the approximations are highly precise, as shall be seen from the numerical examples shown in the sequel.

V. Numerical Results and Discussions

In this Section, the exact LCR and AFD expressions obtained in previous Sections are plotted for some representatives cases. The expressions are validated by Monte Carlo simulation. The closed-form approximations for EGC and MRC are also shown in the figures, and a very good agreement is observed.

Fig. 1 depicts the LCR and AFD of PSC, EGC, and MRC over i.i.d. α-μ fading channels, for different values of the parameters α_i and μ_i. Two-branch diversity is considered. The curves with no diversity have been omitted for the sake of clarity. Of course, as well known, the use of diversity reduces drastically the deleterious effect of fading. Note that EGC and MRC present similar performances, and that PSC exhibits the worst performance.

Comparisons between our exact analytical expressions and simulation results are given in Fig. 2. Note the excellent agreement in all of the examples given. A myriad of other cases has been exhaustively investigated, and a very good match has been observed in all of them.

The high accuracy of the closed-form approximations proposed here for EGC and MRC is illustrated in Figs. 3 and 4 considering four-branch diversity. Fig. 3 (EGC case), for $\alpha_i = 1.5$, and Fig. 4 (MRC case), for $\alpha_i = 2.5$, are plotted for different values of the parameter μ_i. We note that for both diversity schemes exact and approximate curves are practically coincident.

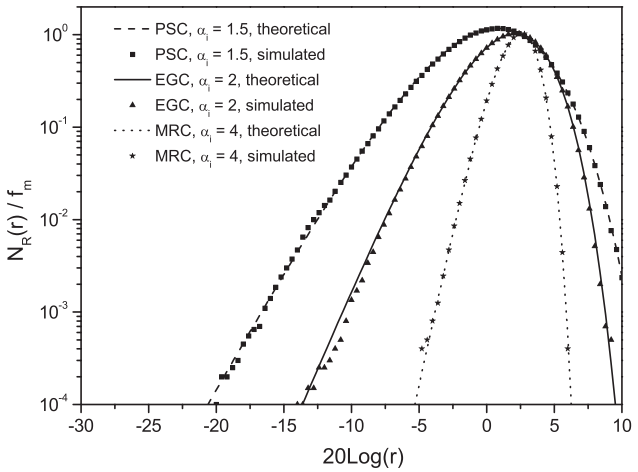

Fig. 2. Comparison between simulated and theoretical curves for PSC, EGC, and MRC with $M = 2$, $\Omega_i = 1$, $\mu_i = 2$, and varying α_i.

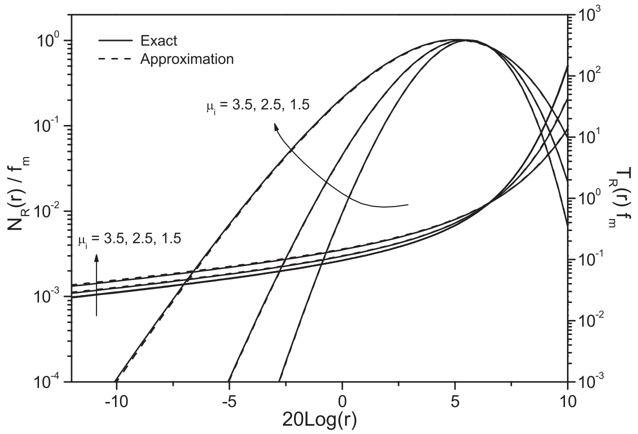

Fig. 3. LCR and AFD of EGC technique for α-μ fading channels ($M = 4$, $\Omega_i = 1$, $\alpha_i = 1.5$, and varying μ_i).

It is noteworthy that our *approximate* solutions are, in fact, *exact* solutions for $\alpha_i = 1$ in EGC, and $\alpha_i = 2$ in MRC. Exhaustive tests show that even for values greatly departing from these, the approximations are still excellent.

VI. Conclusions

Exact expressions for LCR and AFD of PSC, EGC, and MRC operating over independent non-identical α-μ fading channels have been presented. In the EGC and MRC cases, for which the exact solutions are in multifold integral form, simple and accurate closed-form approximations have been derived. The exact analytical formulas have been validated by specializing them to known particular cases and, more generally, by means of simulation. Our derivations find applicability in the analysis and design of wireless diversity systems over generalized fading conditions, in which both non-linearity and clusterization phenomena occur.

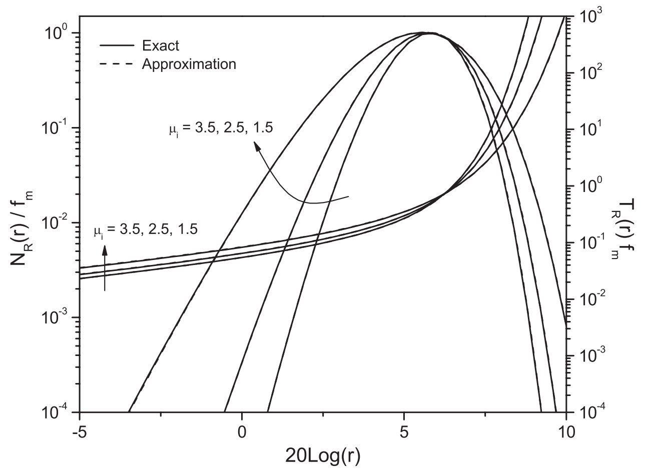

Fig. 4. LCR and AFD of MRC technique for α-μ fading channels ($M = 4$, $\Omega_i = 1$, $\alpha_i = 2.5$, and varying μ_i).

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